## Chapter 2: Rational Expressions and Equations

## Section 2.1: Introduction to Rational Expressions

\#1-8: Substitute the given number into the expression and simplify (if possible).

1) $\frac{2}{x+3}$ let $x=0$
2) $\frac{3}{2 x+5}$ let $x=0$
3) $\frac{x-3}{x^{2}+4 x+1}$ let $x=3$
4) $\frac{x^{2}-16}{x+1}$ let $x=4$
5) $\frac{(a-1)(a+2)}{(a-3)(a-4)}$ let $a=3$
6) $\frac{(b-2)(b+1)}{(b-6)(2 b-5)}$ let $b=6$
7) $\frac{2}{x-3}$ let $x=3$
8) $\frac{3}{2 x}$ let $x=0$
\#9-16: Write the domain of the expression using words and in interval notation.
9) $\frac{3}{h-5}$
10) $\frac{4}{x+7}$
11) $\frac{x+1}{x-2}$
12) $\frac{x-3}{x-4}$
13) $\frac{y-4}{y^{2}+5 y+6}$
14) $\frac{y^{2}-9}{y^{2}-6 y-7}$
15) $\frac{a-3}{a^{2}+6 a-16}$
16) $\frac{b^{2}-2}{b^{2}+9 b+20}$
\#17-30: Reduce the expression to lowest terms.
17) $\frac{8 a^{2}}{16 a}$
18) $\frac{25 x^{4}}{15 x^{3}}$
19) $\frac{20 x y^{3}}{16 x^{5} y}$
20) $\frac{14 x^{3} t^{2}}{21 x t^{6}}$
21) $\frac{-36 x^{4} y z^{2}}{16 x^{5} y z^{7}}$
22) $\frac{-18 x y^{2} z^{3}}{24 x^{2} y^{2} z}$
23) $\frac{2(a-3)}{12(a-3)}$
24) $\frac{12(y-7)}{36(y-7)}$
25) $\frac{(x-1)(x+4)}{(x-1)(x+2)}$
26) $\frac{(p-3)(p+4)}{(p-6)(p-3)}$
27) $\frac{10(m+3)}{12(m+3)(m-1)}$
28) $\frac{15(x-1)(x+3)}{(x-1)(x+2)(x+3)}$
29) $\frac{10}{5 b-30}$
30) $\frac{12}{24 b+6}$

## Chapter 2: Rational Expressions and Equations

## Section 2.1: Introduction to Rational Expressions

\#31-38: Reduce the expression to lowest terms.
31) $\frac{10 x-20}{x^{2}-4}$
32) $\frac{5 x+15}{x^{2}-9}$
33) $\frac{3 x+18}{x^{2}+5 x-6}$
34) $\frac{5 x-5}{x^{2}-7 x+6}$
35) $\frac{x^{2}-9}{x^{2}+5 x+6}$
36) $\frac{b^{2}+b-6}{b^{2}+2 b-8}$
37) $\frac{2 y^{2}+y-6}{y^{2}+y-2}$
38) $\frac{6 x^{2}-x-1}{2 x^{2}+9 x-5}$
\#39-46: Reduce the expression involving a ratio of (-1).
39) $\frac{x-2}{2-x}$
40) $\frac{y-3}{3-y}$
41) $\frac{-3-2 x}{3+2 x}$
42) $\frac{-3 x-7}{3 x+7}$
43) $\frac{3 x-6}{12-6 x}$
44) $\frac{10-2 x}{3 x-15}$
45) $\frac{25-x^{2}}{x-5}$
46) $\frac{36-x^{2}}{x-6}$

## Chapter 2: Rational Expressions and Equations

## Section 2.2: Multiplication and Division of Rational Expressions

\#1-6: Multiply or divide the fractions.

1) $\frac{3}{10} \cdot \frac{35}{6}$
2) $\frac{8}{9} \cdot \frac{15}{14}$
3) $\frac{3}{10} \div \frac{30}{5}$
4) $\frac{18}{7} \div \frac{30}{21}$
5) $3 \cdot \frac{5}{21}$
6) $\frac{2}{3} \cdot 36$
\#7-33: Multiply or divide as indicated.
7) $\frac{8 a}{4 a-20} \cdot \frac{a-5}{12 a^{2}}$
8) $\frac{5 b+15}{2 b} \cdot \frac{b}{3 b+9}$
9) $\frac{3 x+21}{6 x} \cdot \frac{3 x^{2}}{4 x+28}$
10) $\frac{14 x^{2}}{6 x-42} \cdot \frac{3 x-21}{6 x}$
11) $\frac{3-x}{12} \cdot \frac{14}{x-3}$
12) $\frac{16}{x-2} \cdot \frac{2-x}{8}$
13) $\frac{a^{2}-b^{2}}{a} \cdot \frac{b}{a^{2} b-a b^{2}}$
14) $\frac{y}{x^{2}-y^{2}} \cdot \frac{x^{2}-x y}{x}$
15) $\frac{4 a+16}{3 a-15} \div \frac{a+4}{2 a-10}$
16) $\frac{2 x+10}{5 x-20} \div \frac{3 x+15}{3 x-12}$
17) $\frac{3 b-6}{12 b} \div \frac{10 b-20}{3 b^{2}}$
18) $\frac{5 y}{4 y-12} \div \frac{10 y^{2}}{6 y-18}$
19) $\frac{p^{2}+2 p+1}{4 p-1} \cdot \frac{16 p^{2}-1}{p^{2}-1}$
20) $\frac{x^{2}-9}{3 x+4} \cdot \frac{9 x^{2}-16}{x^{2}+6 x+9}$
21) $\frac{2 a^{2}-a-6}{3 a^{2}-5 a-2} \cdot \frac{3 a^{2}+7 a+2}{2 a^{2}+7 a+6}$
22) $\frac{x^{2}-9 x+20}{x^{2}-5 x+6} \times \frac{x^{2}-3 x+2}{x^{2}-5 x+4}$
23) $\frac{2 x^{2}+5 x-12}{3 x^{2}-8 x-16} \div \frac{2 x^{2}+3 x-9}{3 x^{2}+13 x+12}$
24) $\frac{x^{2}+6 x+5}{x^{2}+4 x+3} \div \frac{x^{2}-25}{x^{2}-9}$
25) $(x+3) \cdot \frac{x}{2 x^{2}+5 x-3}$
26) $(y-4) \cdot \frac{y}{y^{2}+2 y-24}$
27) $\left(x^{2}-2 x-3\right) \div \frac{4 x-12}{x-1}$
28) $\left(4 x^{2}-9\right) \div \frac{4 x+6}{x+3}$
29) $\frac{6-2 x}{2 x+8} \div(9-3 x)$
30) $\frac{x^{2}-25}{2 x+10} \div\left(x^{2}-10 x+25\right)$
31) $\frac{x^{2}+2 x}{5 x} \div\left(2 x^{2}+x-6\right)$
32) $\frac{x^{2}-3 x}{6 x} \div\left(2 x^{2}-5 x-3\right)$
33) $\frac{x^{2}+6 x+5}{x^{2}-1} \cdot \frac{x^{2}+2 x+1}{x^{2}-25}$

## Chapter 2: Rational Expressions and Equations

## SECTION 2.3 has been deleted

## Section 2.4: Addition and Subtraction of Rational Expressions

\#1-14: Add or subtract the expressions with like denominators, simplify as much as possible.

1) $\frac{2}{9}+\frac{1}{9}$
2) $\frac{3}{10}+\frac{2}{10}$
3) $\frac{2}{7}-\frac{-5}{7}$
4) $\frac{3}{5}-\frac{-4}{5}$
5) $\frac{2 b+1}{b+3}+\frac{5}{b+3}$
6) $\frac{4 a-18}{a-3}+\frac{6}{a-3}$
7) $\frac{4 d+3}{d+5}-\frac{d-12}{d+5}$
8) $\frac{5 z+1}{z-4}-\frac{z+17}{z-4}$
9) $\frac{2 x}{x-4}-\frac{x-3}{x-4}$
10) $\frac{2 x-5}{x+1}-\frac{x-3}{x+1}$
11) $\frac{x^{2}}{x-6}-\frac{5 x+6}{x-6}$
12) $\frac{x^{2}}{x+3}-\frac{9}{x+3}$
13) $\frac{b^{2}}{b+2}+\frac{3 b+2}{b+2}$
14) $\frac{a^{2}}{a-1}+\frac{3 a-4}{a-1}$
\#15-32: Add or subtract the expressions with unlike denominators, simplify as much as possible.
15) $\frac{3}{a b^{2}}+\frac{2}{a b}$
16) $\frac{7}{y z}+\frac{6}{y^{2} z}$
17) $\frac{3}{2 a b}-\frac{5}{a b}$
18) $\frac{5}{6 x y}-\frac{2 y}{9 x}$
19) $\frac{2 w-1}{2 w-6}-\frac{8-w}{w-3}$
20) $\frac{2 x}{x+4}-\frac{x-4}{5 x+20}$
21) $\frac{4 b}{3 b-12}+\frac{5 b-1}{2 b-8}$
22) $\frac{6 y+1}{3 y-21}+\frac{2 y-1}{4 y-28}$
23) $\frac{7}{m+5}-\frac{2 m-3}{m^{2}-25}$
24) $\frac{2}{m-4}-\frac{m-2}{m^{2}-16}$
25) $\frac{5}{2 x^{2}+3 x}+\frac{7}{4 x+6}$
26) $\frac{x+3}{3 x^{2}+5 x}+\frac{2}{3 x+5}$
27) $\frac{4 x}{x-3}-\frac{12-x}{3-x}$
28) $\frac{3 y}{2-y}-\frac{6-2 y}{y-2}$
29) $\frac{3}{a^{2}-7 a-18}+\frac{5}{a^{2}-4}$
30) $\frac{2}{b^{2}+5 b+4}+\frac{6}{b^{2}-16}$
31) $\frac{2}{y^{2}-3 y-4}-\frac{5}{y^{2}-16}$
32) $\frac{6}{a^{2}+2 a+1}-\frac{1}{a^{2}-3 a-4}$

## Chapter 2: Rational Expressions and Equations

## Section 2.4: Addition and Subtraction of Rational Expressions

\#33-44: Add or subtract the expressions with unlike denominators, simplify as much as possible.
33) $\frac{x}{x+y}-\frac{2}{x^{2}-y^{2}}$
34) $\frac{3 x}{x^{2}+3 x y+2 y^{2}}-\frac{4}{x+2 y}$
35) $\frac{2 y-1}{4 y+1}+\frac{2 y-3}{4 y^{2}+9 y+2}$
36) $\frac{2 z}{2 z^{2}-z-3}+\frac{z-3}{z+1}$
37) $\frac{a-1}{9 a^{2}+6 a-8}-\frac{3}{3 a^{2}-2 a-8}$
38) $\frac{m+3}{8 m^{2}+15 m-2}-\frac{2}{5 m^{2}+6 m-8}$
39) $\frac{b+4}{12 b^{2}-5 b-2}+\frac{b-1}{3 b^{2}-17 b+10}$
40) $\frac{b+1}{2 b^{2}+9 b-5}+\frac{b+2}{4 b^{2}+23 b+15}$
41) $\frac{3}{x-1}+5$
42) $\frac{2}{x-3}+4$
43) $\frac{1}{x+1}+3 x$
44) $\frac{2}{x+5}+x$

## Chapter 2: Rational Expressions and Equations

## Section 2.5: Rational Equations

\#1-22: Solve the rational equations. Be sure to check all solutions. If a solution does not check state that it is extraneous.

1) $\frac{3}{x}+\frac{5}{6}=\frac{2}{x}$
2) $\frac{14}{3 x}-\frac{5}{x}=\frac{-1}{6}$
3) $5-\frac{2}{x}=\frac{16}{2 x}$
4) $\frac{7}{x}-2=\frac{1}{4}$
5) $\frac{2}{y}+1=\frac{3}{y^{2}}$
6) $1-\frac{2}{x}=\frac{8}{x^{2}}$
7) $\frac{x}{3}+\frac{1}{x}=\frac{7}{6}$
8) $\frac{y}{2}+\frac{2}{y}=\frac{10}{3}$
9) $\frac{x}{5}+\frac{x+2}{x}=\frac{2}{x}$
10) $\frac{x}{3}-\frac{x+1}{x}=\frac{5}{x}$
11) $\frac{w}{5}-\frac{w+3}{w}=\frac{-3}{w}$
12) $\frac{x}{2}-\frac{x+2}{2}=\frac{-2}{x}$
13) $\frac{2}{x+3}=\frac{2}{2 x+6}+1$
14) $\frac{3}{x+4}=\frac{2}{4 x+16}+\frac{5}{2}$
15) $\frac{x}{x-2}+\frac{1}{x+4}=\frac{x-8}{x-2}$
16) $\frac{2 x}{x+3}=1-\frac{6}{x+3}$
17) $\frac{5}{x-3}=\frac{2 x}{x^{2}-9}$
18) $\frac{3}{x+1}=\frac{2 x}{x^{2}-1}$
19) $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{x^{2}-6 x+8}$
20) $\frac{x}{x+1}+\frac{2}{x-3}=\frac{2}{x^{2}-2 x-3}$
21) $\frac{3}{x-3}=\frac{x}{x-3}-\frac{3}{2}$
22) $\frac{2}{x}-\frac{4}{x+1}=3$

## Chapter 2: Rational Expressions and Equations

## Section 2.5: Rational Equations

\# 23-30: Solve the following proportions.
23) $\frac{5}{x}=\frac{3}{12}$
26) $\frac{7}{5}=\frac{x}{10}$
29) $\frac{9}{3 x-1}=\frac{2}{x}$
30) $\frac{5}{2 x-3}=\frac{3}{x}$

## Chapter 2: Rational Expressions and Equations

## Section 2.6: Applications of Rational Equations and Proportions

1) A lawn fertilizer calls for 2 pounds for 150 square feet. At this rate, how many pounds are required for 825 square feet?
2) A sample of 5 new cars is selected; it is found that 2 of the cars have cosmetic imperfections. At this rate, how many cars out of 1,000 will be expected to have cosmetic imperfections?
3) A map has a scale of 20 miles equals 1 inch. How many inches will two cities be apart on the map if they are actually 140 miles apart?
4) 1 out of every 4 lottery scratcher's tickets is a winner. A person purchases 20 scratcher's tickets. How many of them should be winning tickets?
5) Jess answered 8 of 10 trivia questions correct. At this rate how many questions out of 250 will she answer correctly?
6) 1 of every 13 people has red hair. How many people in a group of 200 people would you expect to have red hair? (Round to the nearest person)
7) A boat travels 10 miles upstream against the current in the same time it takes to go 30 miles downstream with the current. If the speed of the current is 2 miles per hour, find the speed of the boat in still water.
8) A boat travels 32 miles downstream with the current in the same time it goes 8 miles upstream against the current. If the speed of the boat is 5 miles per hour in still water, find the speed of the current.
9) A plane flies 400 miles with the wind in the same time it takes to fly 320 miles against the wind. If the speed of the wind is 20 miles per hour, find the speed of the plane in still air.
10. A plane flies 520 miles with the wind in the same time it travels 480 miles against the wind. The speed of the plane in still air is 250 miles per hour. Find the speed of the wind.
11. One person runs 3 miles per hour slower than another. The faster runner can cover 15 miles in the same time the other can run 6 miles. Find the speed of each runner.
12. One person walks 1 mile per hour faster than the other. The slower walker travels 10 miles in the same time the faster walker travels 12 miles. Find the speed of each walker.

## Chapter 2: Rational Expressions and Equations

Extra practice for section 2.6

1) A boat travels 24 miles upstream against the current in the same time it takes to go 48 miles downstream with the current. If the speed of the current is 4 miles per hour, find the speed of the boat in still water. (answer speed of boat is 12 mph ).

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| With current |  |  |  |
| Against current |  |  |  |

2) A boat travels 28 miles upstream against the current in the same time it takes to go 52 miles downstream with the current. If the speed of the current is 3 miles per hour, find the speed of the boat in still water. (answer speed of boat is 10 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| With current |  |  |  |
| Against current |  |  |  |

3) A boat travels 36 miles upstream against the current in the same time it takes to go 44 miles downstream with the current. If the speed of the current is 2 miles per hour, find the speed of the boat in still water. (answer speed of boat is 20 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| With current |  |  |  |
| Against current |  |  |  |

4) A boat travels 57 miles downstream with the current in the same time it goes 39 miles upstream against the current. If the speed of the boat is 16 miles per hour in still water, find the speed of the current. (answer speed of current is 3 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| With current |  |  |  |
| Against current |  |  |  |

## Chapter 2: Rational Expressions and Equations

Extra practice for 2.6
5) A boat travels 54 miles downstream with the current in the same time it goes 46 miles upstream against the current. If the speed of the boat is 25 miles per hour in still water, find the speed of the current. (answer speed of current is 2 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| With current |  |  |  |
| Against current |  |  |  |

6) One person runs 2 miles per hour slower than another. The faster runner can cover 12 miles in the same time the other can run 8 miles. Find the speed of each runner. (answer fast person 6 mph , slow 4 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| Fast person |  |  |  |
| Slow person |  |  |  |

7) One person runs 3 miles per hour slower than another. The faster runner can cover 24 miles in the same time the other can run 15 miles. Find the speed of each runner. (answer fast person 8 mph , slow 5 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| Fast person |  |  |  |
| Slow person |  |  |  |

8) One person rides a bike 2 mile per hour faster than the other. The slower rider travels 20 miles in the same time the faster rider travels 24 miles. Find the speed of each rider. (answer fast person 12 mph , slow 10 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| Fast person |  |  |  |
| Slow person |  |  |  |

## Chapter 2: Rational Expressions and Equations

Extra practice for 2.6
9) One person rides a bike 3 mile per hour faster than the other. The slower rider travels 24 miles in the same time the faster rider travels 36 miles. Find the speed of each rider. (answer fast person 9 mph , slow 6 mph )

|  | distance | rate | time |
| :--- | :--- | :--- | :--- |
| Fast person |  |  |  |
| Slow person |  |  |  |

## Chapter 2: Rational Expressions and Equations

## Section 2.7: Review

1. Find the domain of the given expression; write your answer in a sentence.
a) $\frac{x+2}{x-4}$
b) $\frac{b^{2}-4}{b^{2}-2 b-63}$
2. Reduce the rational expression to lowest terms.
a) $\frac{24 a^{4} b^{3}}{21 a b^{5}}$
b) $\frac{2 x-16}{x^{2}-64}$
c) $\frac{x-3}{x^{2}-5 x+6}$
3. Multiply or divide as indicated.
a) $\left(x^{2}+5 x+6\right) \div \frac{3 x^{2}+4 x-4}{3 x-2}$
b) $\frac{y^{2}-3 y+2}{3 y^{2}-12} \cdot \frac{y^{2}+6 y+8}{y^{2}+5 y+4}$
c) $\frac{b^{2}+5 b+4}{3 b^{2}+8 b+5} \div \frac{b^{2}-16}{3 b^{2}+2 b-5}$
4. Add or subtract as indicated.
a) $\frac{5}{x-4}+\frac{3}{5 x-20}$
b) $\frac{2 x+3}{x-5}+\frac{x-18}{x-5}$
c) $\frac{5 m+3}{m^{2}-9}-\frac{4}{m-3}$
d) $\frac{7}{9 b}-\frac{3}{4 b^{2}}$
e) $\frac{6 x}{3 x^{2}+2 x-1}+\frac{5}{x^{2}-1}$
5) Solve the rational equations. Be sure to check all solutions. If a solution does not check state that it is extraneous. (Some of these problems are taken from section 2.5 , you may have done them already.)
a) $\frac{x-4}{4}+\frac{x}{3}=6$
b) $\frac{6}{x}-\frac{9}{x-1}=\frac{1}{4}$
c) $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{x^{2}-6 x+8}$
d) $1-\frac{2}{x}=\frac{8}{x^{2}}$
e) $\frac{x+1}{2 x}=\frac{3}{5}$
f) $\frac{5}{2 x-3}=\frac{3}{x}$
6) A map has a scale of 50 miles equals 2 inches. How many inches will two cities be apart on the map if they are actually 3,000 miles apart?

## Chapter 2: Rational Expressions and Equations

## Section 2.7: Review

7) A boat travels 24 miles upstream against the current in the same time it takes to go 48 miles downstream with the current. If the speed of the current is 4 miles per hour, find the speed of the boat in still water.
8) One person runs 1 mile per hour slower than another. The faster runner can cover 20 miles in the same time the other can run 16 miles. Find the speed of each runner.

## Chapter 2: Rational Expressions and Equations

## Grima, Mat 120 chapter 2 Practice test

1. Multiply or divide as indicated.
a) $\frac{n^{2}+2 n-3}{n^{2}+4 n-5} * \frac{n^{2}-3 n-10}{n^{2}+5 n+6}$
b) $\frac{m-2}{m+3} * \frac{m^{2}+6 m+9}{m^{2}-4}$
c) $\frac{x^{2}-x-6}{x^{2}-x-12} \div \frac{x^{2}+3 x+2}{x^{2}-3 x-4}$
2. Find the domain of the given expression; write your answer in a sentence.

$$
\frac{x+2}{2 x+20}
$$

3) Solve the rational equations. Be sure to check all solutions. If a solution does not check state that it is extraneous
a) $\frac{x}{3}-\frac{x}{4}=\frac{1}{6}$
b) $\frac{x}{x+5}-\frac{5}{x-5}=\frac{14}{x^{2}-25}$ (hint
$13 * 3=39$ )
c) $\frac{3}{x}=\frac{4}{x-10}$
4) Reduce the rational expression to lowest terms.
a) $\frac{14 x^{2}}{16 x^{7}}$
b) $\frac{3 x^{2}-8 x+4}{x^{2}-5 x+6}$
c) $\frac{x^{2}+3 x-4}{x^{2}+6 x+8}$
5) Add or subtract as indicated.
a) $\frac{3 x}{x+6}+\frac{18}{x+6}$
b) $\frac{2 x-5}{x+3}+\frac{16}{x+3}$
c) $\frac{2}{x^{2}+6 x+5}+\frac{3}{x^{2}+3 x+2}$
6) One person runs 2 miles per hour slower than another. The faster runner can cover 15 miles in the same time the other can run 9 miles. Find the speed of each runner.
7) A boat travels 15 miles upstream against the current in the same time it takes to go 45 miles downstream with the current. If the speed of the current is 5 miles per hour, find the speed of the boat in still water.

Answers:1a) $\frac{n-5}{n+5} \begin{array}{llll}\text { 1b) } \frac{m+3}{m+2} & \text { 1c) } \frac{x-3}{x+3} & \text { 2) The domain is all real numbers except } \mathrm{x}=-10\end{array}$
2) okay to write $(-\infty,-10) \cup(-10, \infty)$ also okay to write shorthand answer
3a) $x=2$
3b) $x=-3$ and $x=13$
$3 \mathrm{c}) \mathrm{x}=-30$
4a) $\frac{7}{8 x^{5}}$
4b) $\frac{3 x-2}{x-3}$
4c) $\begin{array}{ll}\frac{x-1}{x+2} & \text { 5a) } 3\end{array}$
5b) $\frac{2 x+11}{x+3}$
5c) $\frac{5 x+19}{(x+1)(x+2)(x+5)}$
6) fast runner 5 mph slow runner 3 mph 7 ) boat speed 10 mph

Chapter 2: Rational Expressions and Equations

