Chapter 2: Rational Expressions and Equations
Solutions to Selected Odd Problems

Section 2.1

1) Substitute the given number into the expression and simplify (if possible).

$\frac{2}{x+3}$ $let\ x = 0$

Do this by replacing x with 0 and simplifying

$\frac{2}{0+3} = \frac{2}{3}$

Solution: $\mathbf{\frac{2}{3}}$

3) Substitute the given number into the expression and simplify (if possible)

$\frac{x-3}{x^2+4x+1}$ $let\ x = 3$

Do this by replacing x with 3 and simplifying

$\frac{3-3}{3^2+4\cdot 3+1} = \frac{0}{9+12+1} = \frac{0}{13} = 0$

(any fraction with a 0 in the numerator is equal to zero)

Solution: 0

5) Substitute the given number into the expression and simplify (if possible)

$\frac{(a-1)(a+2)}{(a-3)(a-4)}$ $let\ a = 3$

Do this by replacing a with 3 and simplifying

$\frac{(3-1)(3+2)}{(3-3)(3-4)} = \frac{2\cdot 5}{0\cdot -1} = \frac{10}{0} = undefined$

(any fraction with 0 in the denominator is undefined)

Solution: Undefined

11) Write the domain of the expression using words and in interval notation

$\frac{x+1}{x-2}$

To find the domain, ignore the numerator, find where the denominator equals zero, then write your answer

x-2 = 0

Solve this to get x = 2, this number must be excluded from the domain. Each of the forms of the answer exclude 2 from the domain.

Solution: domain = $(-\infty, \mathbf{2}) \cup (\mathbf{2}, \infty)$
Domain: All real numbers except x = 2

Chapter 2: Rational Expressions and Equations
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Section 2.1

15) Write the domain of the expression using words and in interval notation

$\frac{a-3}{a^2+6a-16}$

To find the domain, ignore the numerator, find where the denominator equals zero, then write your answer.

$a^2 + 6a - 16 = 0$

$(a+8)(a-2)=0$

$a+8=0, \quad a-2=0$

$a = -8$ or $a = 2$

The numbers -8 and 2 must be excluded from the domain. Each form of the answer excludes 2 and -8.

Solution: domain = $(-\infty, -8) \cup (-8, 2) \cup (2, \infty)$
Domain all real numbers except x = -8, 2

19) Reduce the expression to lowest terms

$\frac{20xy^3}{16x^5y}$

NUMBERS: The fraction $\frac{20}{16}$ reduces by 4 to: $\frac{5}{4}$

X's: Subtract the exponents on the x's and leave the x where the larger exponent is. The x will remain in the denominator, because the larger exponent is in the denominator.

$x^{5-1} = x^4$ (in the denominator)

Y's: Subtract the exponents of the y's and leave the y in the numerator, because the larger exponent is in the numerator.
$y^{3-1} = y^2$ (in the numerator)

Write your answer.

Solution: $\frac{5y^2}{4x^4}$

31) Reduce the expression to lowest terms
$\frac{10x-20}{x^2-4}$

First factor both the numerator and the denominator:
$= \frac{10(x-2)}{(x+2)(x-2)}$

Cancel the $(x-2)$'s and write your answer.

Solution: $\frac{10}{x+2}$

35) Reduce the expression to lowest terms

$\frac{x^2-9}{x^2+5x+6}$

First factor both the numerator and the denominator:

$= \frac{(x+3)(x-3)}{(x+2)(x+3)}$

Cancel the (x+3)'s and write your answer.

Solution: $\frac{x-3}{x+2}$

Chapter 2: Rational Expressions and Equations
Solutions to Selected Odd Problems

Section 2.1

39) Reduce the expression to lowest terms

$\frac{x-2}{2-x}$

Rewrite the denominator with the x first

$= \frac{x-2}{-x+2}$

Then factor out a (-1) from the denominator.

$= \frac{x-2}{-1(x-2)}$

Cancel the (x-2)'s and you are left with

$= \frac{1}{-1}$

Write your answer.

Solution: -1

43) Reduce the expression to lowest terms

$\frac{3x-6}{12-6x}$

Rewrite the denominator with the 6x first.

$= \frac{3x-6}{-6x+12}$

Factor out 3 from the numerator, and (-6) from the denominator

$= \frac{3(x-2)}{-6(x-2)}$

NUMBERS: reduce $\frac{3}{-6}$ to get $-\frac{1}{2}$

(x-2)'s **:** These cancel, and there is no x-2 in the final answer.

Write your answer.

Solution: $-\frac{\mathbf{1}}{\mathbf{2}}$

Chapter 2: Rational Expressions and Equations
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Section 2.2

7) Multiply or divide as indicated

$\frac{8a}{4a-20} \cdot \frac{a-5}{12a^2}$

Factor the left denominator
$=\frac{8a}{4(a-5)} \cdot \frac{a-5}{12a^2}$

Reduce the 4 in the denominator with the 8 in the numerator to get
$=\frac{2a}{a-5} \cdot \frac{a-5}{12a^2}$

Reduce the 2 in the numerator, with the 12 in the denominator

Cancel the (a – 5)'s

Subtract the exponents on the a's, leave the a in the denominator because that is where the larger exponent is.

$=\frac{1}{1} \cdot \frac{1}{6a}$
Write your answer.

Solution: $\mathbf{\frac{1}{6a}}$

9) Multiply or divide as indicated
$\frac{3x+21}{6x} \cdot \frac{3x^2}{4x+28}$
First factor.

$= \frac{3(x+7)}{6x} \cdot \frac{3x^2}{4(x+7)}$

Reduce the 6 in the left denominator with the 3 in the right numerator.

Reduce the x in the left denominator with the x in the right numerator, but subtracting exponents, leave the x in the numerator because that is where the larger exponent is.
x^{2-1} = x (in the numerator)

Cancel the (x +7)'s

Now you are left with:

$\mathbf{= \frac{3}{2} \cdot \frac{x}{4}}$

Multiply, and write answer

Solution: $\mathbf{\frac{3x}{8}}$

Chapter 2: Rational Expressions and Equations
Solutions to Selected Odd Problems

Section 2.2

11) Multiply or divide as indicated

$\frac{3-x}{12} \cdot \frac{14}{x-3}$

Write the 3-x in the left numerator with the x first.
$= \frac{-x+3}{12} \cdot \frac{14}{x-3}$

Factor out a (-1) from the left numerator.

$= \frac{-1(x-3)}{12} \cdot \frac{14}{x-3}$

Reduce the 14 and 12,

Cancel the (x − 3)'s
$= \frac{-1}{6} \cdot \frac{7}{1}$

Multiply, and write answer.

Solution: $-\frac{7}{6}$

15) Multiply or divide as indicated
$\frac{4a+16}{3a-15} \div \frac{a+4}{2a-10}$

Rewrite as a multiplication problem

$= \frac{4a+16}{3a-15} \cdot \frac{2a-10}{a+4}$

Factor

$= \frac{4(a+4)}{3(a-5)} \cdot \frac{2(a-5)}{a+4}$

Cancel the (a+4)'s and (a − 5)'s

$= \frac{4}{3} \cdot \frac{2}{1}$

Multiply, and write answer.

Solution: $\frac{8}{3}$

19) Multiply or divide as indicated
$\frac{p^2+2p+1}{4p-1} \cdot \frac{16p^2-1}{p^2-1}$

Factor

$= \frac{(p+1)(p+1)}{4p-1} \cdot \frac{(4p+1)(4p-1)}{(p+1)(p-1)}$

Cancel (4p-1)'s
Cancel one of the (p+1)'s in the left numerator with the (p-1) in the right denominator.

$= \frac{p+1}{1} \cdot \frac{4p+1}{p-1}$
Multiply, and write answer.
(it would be o.k. if you foiled out the numerator)

Solution: $\frac{(p+1)(4p+1)}{p-1}$

23) multiply or divide as indicated
$\frac{2x^2+5x-12}{3x^2-8x-16} \div \frac{2x^2+3x-9}{3x^2+13x+12}$

Rewrite as a multiplication problem

$= \frac{2x^2+5x-12}{3x^2-8x-16} \cdot \frac{23+13x+12}{32+3x-9}$

Factor

$= \frac{(2x-3)(x+4)}{(3x+4)(x-4)} \cdot \frac{(3x+4)(x+3)}{(2x-3)(x+3)}$

Cancel and write your answer.

Solution: $\frac{x+4}{x-4}$

Chapter 2: Rational Expressions and Equations
Solutions to Selected Odd Problems

Section 2.2

27) Multiply or divide as indicated

$(x^2 - 2x - 3) \div \frac{4x-12}{x-1}$

Rewrite as a multiplication problem, I will also write the $x^2 - 2x - 3$ as a fraction , but it is necessary to do this.

$= \frac{x^2-2x-3}{1} \cdot \frac{x-1}{4x-12}$

Factor

$= \frac{(x+1)(x-3)}{1} \cdot \frac{x-1}{4(x-3)}$

Cancel

$= \frac{x+1}{1} \cdot \frac{x-1}{4}$

Multiply, and write answer. (it would be o.k. if you foiled out the numerator)

Solution: $\frac{(x+1)(x-1)}{4}$

31) Multiply or divide as indicated

$\frac{x^2+2x}{5x} \div (2x^2 + x - 6)$

Rewrite as a multiplication problem

$= \frac{x^2+2x}{5x} \cdot \frac{1}{2x^2+x-6}$

Factor

$= \frac{x(x+2)}{5x} \cdot \frac{1}{(2x-3)(x+2)}$

Cancel

$= \frac{1}{5} \cdot \frac{1}{2x-3}$

Multiply, and write answer. (we usually do not multiply out denominators)

Solution: $\frac{1}{5(2x-3)}$

Chapter 2: Rational Expressions and Equations
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Section 2.3

5) $\frac{2}{8xy^2} = \frac{\quad}{24x^2y^5}$

determine what to multiply the right fraction by, by dividing the denominators

$\frac{24x^2y^5}{8xy^2} = 3xy^3$

Multiply $\frac{2}{8xy^2} \cdot \frac{3xy^3}{3xy^3} = \frac{6xy^3}{24x^2y^5}$

Solution: 6xy^3

9) $\frac{x+2}{x-3} = \frac{\quad}{(x-3)(x-4)}$

determine what to multiply the right fraction by, by dividing the denominators

$\frac{(x-3)(x-4)}{x-3} = x - 4$

Multiply $\frac{x+2}{x-3} \cdot \frac{x-4}{x-4} = \frac{(x+2)(x-4)}{(x-3)(x-4)}$

(we would never foil the denominator of the answer, but it is common to foil the numerator.)

Solution: (x+2)(x – 4) or x^2 – 2x – 8

11) $\frac{2}{3-x} = \frac{\quad}{x-3}$

This problem is a little different because the denominators are so similar.

Divide to determine what to multiply by.

$\frac{x-3}{3-x} = \frac{x-3}{-x+3} = \frac{x-3}{-1(x-3)} = \frac{1}{-1} = -1$

Multiply: $\frac{2}{3-x} \cdot \frac{-1}{-1} = \frac{-2}{-3+x} = \frac{-2}{x-3}$

Solution: -2

15) identify the LCD

$\frac{7}{10}, \frac{3}{14}$

Find the prime factorization of each denominator.

10 = 2*5
14= 2*7
LCD = product of each prime factor, to the highest power it occurs.

LCD = 2*5*7 = 70 (I don't need to use the 2 twice)

Solution: LCD = 70

Chapter 2: Rational Expressions and Equations
Solutions to Selected Odd Problems

Section 2.3

17) Identify the LCD

$\frac{1}{4}, \frac{3}{-4}$

When the denominators are opposites, (same number with different signs) either denominator can be considered the LCD. There is no algebra needed to find the LCD.

It would be correct to say LCD = -4, but I chose 4. It is more common to pick the positive number for the LCD in this kind of problem.

Solution: LCD = 4

19) Identify the LCD

$\frac{1}{4x^2y}, \frac{2}{6xy^2}$

NUMBERS: Find the LCM between 4 and 6. I could do this by writing prime factorizations, but it is easy from me to determine LCM between 4 and 6 is 12.

X's: The LCD needs to have the highest power of x that occurs in either denominator. In this case the LCD will have an x^2.

Y's: The LCD needs to have the highest power of y that occurs in either denominator. In this case the LCD will have a y^2.

THUS the LCD = $12x^2y^2$

Solution: LCD = $12x^2y^2$

23) identify the LCD

$\frac{x}{(x+1)(x-3)}, \frac{5}{(x-3)(x-4)}$

The LCD needs to have one of every factor that occurs in either denominator. If there is a duplicate factor between the denominators, it only needs to be included once in the LCD. If factors have exponents other than 1, the LCD needs to have the highest exponent of that factor.

In this problem the LCD = $(x+1)(x-3)(x-4)$

It would be wrong to write $(x-3)^2$ in the LCD even though it occurs in both denominators. The only time I would have an $(x-3)^2$ as part of the LCD is if there was an $(x-3)^2$ in one of the denominators.

Solution: LCD = $(x+1)(x-3)(x-4)$

25) identify the LCD

$\frac{2}{x-1}, \frac{3}{x-2}$

The LCD needs to have one of every factor that occurs in either denominator. If there is a duplicate factor between the denominators, it only needs to be included once in the LCD. If factors have exponents other than 1, the LCD needs to have the highest exponent of that factor.

In this problem the LCD =$(x-1)(x-2)$

Solution: LCD = $(x-1)(x-2)$

Chapter 2: Rational Expressions and Equations
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Section 2.3

29) Identify the LCD

$\frac{2x}{x^2-9}, \frac{3}{x^2+4x+3}$

I have to factor before I can find the LCD in this problem.

$\frac{2x}{(x+3)(x-3)}, \frac{3}{(x+3)(x+1)}$

The LCD needs to have one of every factor that occurs in either denominator. If there is a duplicate factor between the denominators, it only needs to be included once in the LCD.

Solution: LCD = (x+3)(x – 3)(x+1)

31) Identify the LCD

$\frac{1}{5-x}, \frac{2}{x-5}$

I have to factor before I can find the LCD in this problem. I will factor the left denominator by writing the –x first, then factoring out a (-1)

$\frac{1}{5-x} = \frac{1}{-x+5} = \frac{1}{-(x-5)}$

So the problem is really asking me to find the LCD between

$\frac{1}{-(x-5)}, \frac{2}{x-5}$

Because these are opposites, there is no algebra needed to find the LCD. Either denominator can be considered the LCD. In this case I usually choose the “positive” denominator for the LCD>

Solution: LCD = x – 5

33) $\frac{2}{3y^2}, \frac{2}{5y}$

I won’t explain how to find the LCD, look at the between #15- 31 to see how it can be found.
To figure out what to multiply by, look at the problems between #1-11.

Solution: LCD = $15y^2$

$\frac{2}{3y^2} \cdot \frac{5}{5} = \frac{10}{15y^2}$ $\qquad$ $\frac{2}{5y} \cdot \frac{3y}{3y} = \frac{6y}{15y^2}$

25) identify the LCD $\frac{2}{x-1}, \frac{3}{x-2}$

The LCD needs to have one of every factor that occurs in either denominator. If there is a duplicate factor between the denominators, it only needs to be included once in the LCD. If factors have exponents other than 1, the LCD needs to have the highest exponent of that factor.

In this problem the LCD =(x-1)(x-2)

Solution: LCD = (x-1)(x-2)

$\mathbf{\frac{2}{3y^2} = \frac{10}{15y^2}, \quad \frac{2}{5y} = \frac{6y}{15y^2}}$

Chapter 2: Rational Expressions and Equations
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Section 2.3

37) $\frac{5}{4ab^2}, \frac{6b}{3ab}$

I won't explain how to find the LCD, look at the between #15- 31 to see how it can be found.
To figure out what to multiply by, look at the problems between #1-11.

LCD = $12ab^2$

$\frac{5}{4ab^2} \cdot \frac{3}{3} = \frac{15}{12ab^2}$ $\quad$ $\frac{6b}{3ab} \cdot \frac{4b}{4b} = \frac{24b^2}{12ab^2}$

Solution: LCD = $12ab^2$
$\frac{5}{4ab^2} = \frac{15}{12ab^2}, \quad \frac{6b}{3ab} = \frac{24b^2}{12ab^2}$

41) $\frac{6}{(x+3)(x-4)}, \frac{5}{(x-4)(x+1)}$

I won't explain how to find the LCD, look at the between #15- 31 to see how it can be found.
To figure out what to multiply by, look at the problems between #1-11.

LCD =(x+3)(x-4)(x+1)

$\frac{6}{(x+3)(x-4)} \cdot \frac{x+1}{x+1} = \frac{6x+6}{(x+3)(x-4)(x+1)}$

$\frac{5}{(x-4)(x+1)} \cdot \frac{x+3}{x+3} = \frac{5x+15}{(x+3)(x-4)(x+1)}$

Solution: LCD = (x+3)(x-4)(x+1)

$$\frac{6}{(x+3)(x-4)} = \frac{6x+6}{(x+3)(x-4)(x+1)},$$

$$\frac{5}{(x-4)(x+1)} = \frac{5x+15}{(x+3)(x-4)(x+1)}$$

Chapter 2: Rational Expressions and Equations
Solutions to Selected Odd Problems

Section 2.3

43) $\frac{x}{x^2+6x+5}, \frac{3}{x^2-1}$

First factor the denominators:
$\frac{x}{(x+5)(x+1)}, \frac{3}{(x+1)(x-1)}$
Then determine the LCD = (x+1)(x-1)(x+5)
Then rewrite each fraction with the LCD

$$\frac{x}{(x+5)(x+1)} \cdot \frac{x-1}{x-1} = \frac{x^2-x}{(x+5)(x+1)(x-1)}$$

$$\frac{3}{(x+1)(x-1)} \cdot \frac{x+5}{x+5} = \frac{3x+15}{(x+5)(x+1)(x-1)}$$

Solution: LCD = (x+1)(x-1)(x+5)

$$\boldsymbol{\frac{x}{x^2+6x+5} = \frac{x^2-x}{(x+1)(x-1)(x+5)}}$$

$$, \quad \boldsymbol{\frac{3}{x^2-1} = \frac{3x+15}{(x+1)(x-1)(x+5)}}$$

47) $\frac{1}{b-2}, \frac{3}{2-b}$

First, factor the denominators, do this by writing the right denominator with the b first and factor out a -1

$$\frac{3}{2-b} = \frac{3}{-b+2} = \frac{3}{-1(b-2)}$$

So the problem is asking me to find the LCD between: $\frac{1}{b-2}, \frac{3}{-1(b-2)}$,

These are opposites so either denominator can be considered the LCD.

I will pick LCD = b-2

$$\frac{1}{b-2} = \frac{1}{b-2} \ (this\ fraction\ does\ not\ need\ to\ be\ changed)$$

$$\frac{3}{-1(b-2)} \cdot \frac{-1}{-1} = \frac{-3}{b-2}$$

Solution: LCD = b – 2

$$\frac{1}{b-2} = \frac{1}{b-2}, \ \frac{3}{2-b} = \frac{-3}{b-2}$$

Chapter 2: Rational Expressions and Equations
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Section 2.4

3) $\frac{2}{7} - \frac{-5}{7}$

Since the numerators are equal, you just have to combine the numerators.

$\frac{2}{7} - \frac{-5}{7} = \frac{2-(-5)}{7} = \frac{2+5}{7} = \frac{7}{7} = 1$

Solution: 1

5) $\frac{2b+1}{b+3} + \frac{5}{b+3}$

Since the numerators are equal, you just have to combine the numerators.

$\frac{2b+1}{b+3} + \frac{5}{b+3} = \frac{2b+1+5}{b+3} = \frac{2b+6}{b+3} = \frac{2(b+3)}{b+3}$

Cancel the (b+3)'s and write your answer.

Solution: 2

9) $\frac{2x}{x-4} - \frac{x-3}{x-4}$

Since the numerators are equal, you just have to combine the numerators.

$\frac{2x}{x-4} - \frac{x-3}{x-4} = \frac{2x-(x-3)}{x-4} = \frac{2x-x+3}{x-4} = \frac{x+3}{x-4}$

Solution: $\frac{x+3}{x-4}$

13) $\frac{b^2}{b+2} + \frac{3b+2}{b+2} = \frac{b^2+3b+2}{b+2}$

$= \frac{(b+2)(b+1)}{b+2}$

$= b + 1$

Solution: b+1

Chapter 2: Rational Expressions and Equations
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Section 2.4

17) $\frac{3}{2ab} - \frac{5}{ab}$
I need to find the LCD and write each fraction with the LCD.

LCD = 2ab

$$\frac{3}{2ab} - \frac{5}{ab} = \frac{3}{2ab} - \frac{5}{ab} \cdot \frac{2}{2}$$

$$= \frac{3}{2ab} - \frac{10}{2ab} = \frac{3-10}{2ab} = \frac{-7}{2ab}$$

Solution: $-\frac{7}{2ab}$

21) $\frac{4b}{3b-12} + \frac{5b-1}{2b-8}$

First factor the denominators, and find the LCD

$$= \frac{4b}{3(b-4)} + \frac{5b-1}{2(b-4)}$$

LCD = 6(b-4)

$$\frac{4b}{3(b-4)} + \frac{5b-1}{2(b-4)} =$$

$$\frac{4b}{3(b-4)} \cdot \frac{2}{2} + \frac{5b-1}{2(b-4)} \cdot \frac{3}{3}$$

$$= \frac{8b}{6(b-4)} + \frac{15b-3}{6(b-4)}$$

$$= \frac{8b+15b-3}{6(b-4)} = \frac{23b-3}{6(b-4)}$$

Solution: $\frac{23b-3}{6(b-4)}$

25) $\frac{5}{2x^2+3x} + \frac{7}{4x+6}$

First factor the denominators, and find the LCD

$$= \frac{5}{x(2x+3)} + \frac{7}{2(2x+3)}$$

LCD = 2x(2x+3)

$$= \frac{5}{x(2x+3)} \cdot \frac{2}{2} + \frac{7}{2(2x+3)} \cdot \frac{x}{x}$$

$$= \frac{10}{2x(2x+3)} + \frac{7x}{2x(2x+3)} = \frac{10+7x}{2x(2x+3)}$$

Solution: $\frac{7x+10}{2x(2x+3)}$

29)
$\frac{3}{a^2-7a-18} + \frac{5}{a^2-4}$

First factor the denominators, and find the LCD

$$= \frac{3}{(a-9)(a+2)} + \frac{5}{(a+2)(a-2)}$$

LCD = (a-9)(a+2)(a-2)

$$= \frac{3}{(a-9)(a+2)} \cdot \frac{a-2}{a-2} + \frac{5}{(a+2)(a-2)} \cdot \frac{a-9}{a-9}$$

$$= \frac{3a-6}{LCD} + \frac{5a-45}{LCD} = \frac{8a-51}{(a-9)(a+2)(a-2)}$$

Solution: $\frac{8a-51}{(a-9)(a+2)(a-2)}$

Chapter 2: Rational Expressions and Equations
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Section 2.4

33) $\frac{x}{x+y} - \frac{2}{x^2-y^2}$

First factor the denominators, and find the LCD

$= \frac{x}{x+y} - \frac{2}{(x+y)(x-y)}$

LCD = (x + y)(x − y)

$= \frac{x}{x+y\cdot} \cdot \frac{x-y}{x-y} - \frac{2}{(x+y)(x-y)}$

$= \frac{x^2-xy}{(x+y)(x-y)} - \frac{2}{(x+y)(x-y)} = \frac{x^2-xy-2}{(x+y)(x-y)}$

Solution: $\boldsymbol{\frac{x^2-xy-2}{(x+y)(x-y)}}$

37) $\frac{a-1}{9a^2+6a-8} - \frac{3}{3a^2-2a-8}$

First factor the denominators, and find the LCD

$= \frac{a-1}{(3a+4)(3a-2)} - \frac{3}{(3a+4)(a-2)}$

LCD = (3a − 2)(3a + 4)(a − 2)

$= \frac{a-1}{(3a+4)(3a-2)} \cdot \frac{a-2}{a-2} - \frac{3}{(3a+4)(a-2)} \cdot \frac{3a-2}{3a-2}$

$= \frac{a^2-2a-1a+2}{LCD} - \frac{9a-6}{LCD} = \frac{a^2-2a-1a+2-9a+6}{LCD}$

Solution: $\boldsymbol{\frac{a^2-12a+8}{(3a-2)(3a+4)(a-2)}}$

41) $\frac{3}{x-1} + 5$

LCD = x − 1

$= \frac{3}{x-1} + \frac{5}{1} \cdot \frac{x-1}{x-1}$

$= \frac{3}{x-1} + \frac{5x-5}{x-1} = \frac{3+5x-5}{x-1}$

Solution: $\boldsymbol{\frac{5x-2}{x-1}}$

Section 2.5

3) $5 - \frac{2}{x} = \frac{16}{2x}$

The LCD is 2x, I will multiply each term by the 2x to clear the fractions.

$2x \cdot 5 - 2x \cdot \frac{2}{x} = 2x \cdot \frac{16}{2x}$

$10x - 4 = 16$

$10x = 20$

$x = 2$
(this solution will check, but I will not include the checking of the solution for this problem)

Solution: x = 2

7) $\frac{x}{3} + \frac{1}{x} = \frac{7}{6}$

The LCD is 6x, I will multiply each term by 6x to clear the fractions.

$6x \cdot \frac{x}{3} + 6x \cdot \frac{1}{x} = 6x \cdot \frac{7}{6}$

$2x^2 + 6 = 7x$

$2x^2 - 7x + 6 = 0$

$(2x - 3)(x - 2) = 0$

$2x - 3 = 0 \ \ or \ \ x - 2 = 0$
$x = \frac{3}{2} \ or \ x = 2$
(this solution will check, but I will not include the checking of the solution for this problem)

Solution: $\boldsymbol{x = \frac{3}{2}, 2}$

Chapter 2: Rational Expressions and Equations
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Section 2.5

11) $\frac{w}{5} - \frac{w+3}{w} = \frac{-3}{w}$

The LCD is 5w, I will multiply by 5w to clear the fractions.

$5w \cdot \frac{w}{5} - 5w \cdot \frac{w+3}{w} = 5w \cdot \frac{-3}{w}$

$w^2 - 5(w + 3) = -15$

$w^2 - 5w - 15 = -15$

$w^2 - 5w - 15 + 15 = 0$

$w^2 - 5w = 0$

$w(w - 5) = 0$

$w = 0 \;\; or \; w - 5 = 0$

Solution: w = 5
(w = 0 is extraneous)

This gives potential solutions of w = 0, and w = 5. I will check these because one of the potential solutions does not check.

<table>
<tr>
<td>Check w = 5
$$\frac{5}{5} - \frac{5+3}{5} = \frac{-3}{5}$$
$$\frac{5}{5} - \frac{8}{5} = \frac{-3}{5}$$
$$\frac{-3}{5} = \frac{-3}{5}$$
This is true so w = 5 is a solution</td>
<td>Check w = 0
$$\frac{0}{5} - \frac{0+3}{0} = \frac{-3}{0}$$
The right two fractions are undefined, I can stop checking when this happens. This solution is extraneous.</td>
</tr>
</table>

Chapter 2: Rational Expressions and Equations
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Section 2.5

15) $\frac{x}{x-2}+\frac{1}{x+4}=\frac{x-8}{x-2}$

I will multiply each term by the LCD of (x − 2)(x+4) to clear the fractions.

$(x-2)(x+4)\frac{x}{x-2}+\ (x-2)(x+4)\frac{1}{x+4}=(x-2)(x+4)\frac{x-8}{x-2}$

$(x+4)x\ +(x-2)(1)=(x+4)(x-8)$

$x^2+4x+x-2=x^2-8x+4x-32$

$x^2+5x-2=x^2-4x-32$
(I subtracted x^2 from both sides to get the next line)
$5x-2=-4x-32$

$9x=-30$

$x=-\frac{30}{9}$
(this solution will check, but I will not include the checking of the solution for this problem)

Solution: $\boldsymbol{x=-\frac{30}{9}}$

Chapter 2: Rational Expressions and Equations
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Section 2.5

19) $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{x^2-6x+8}$

I need to factor first. $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{(x-2)(x-4)}$

Now I will multiply by the LCD of (x – 2)(x – 4) to clear the fractions.

$$(x-2)(x-4)\frac{x}{x-2}+(x-2)(x-4)\frac{1}{x-4}=(x-2)(x-4)\frac{2}{(x-2)(x-4)}$$

$$(x-4)(x)+(x-2)(1)=2$$

$$x^2-4x+x-2=2$$

$$x^2-3x-2=2$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$x-4=0, or\ x+1=0$$

This gives potential solutions of $x=4$ *and* $x=-1$

Check solution of x = -1	Check solution of x = 4
$\frac{-1}{-1-2}+\frac{1}{-1-4}=\frac{2}{(-1)^2-6(-1)+8}$ $\frac{-1}{-3}+\frac{1}{-5}=\frac{2}{15}$ $\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$ $\frac{5}{15}-\frac{3}{15}=\frac{2}{15}$ x = -1 is a solution because it checks.	$\frac{4}{4-2}+\frac{1}{4-4}=\frac{2}{4^2-6(4)+8}$ $\frac{4}{2}+\frac{1}{0}=\frac{2}{16-24+8}$ I have a fraction with zero in the denominator, it is undefined and I can stop checking The solution of x = 4 does not check and is extraneous.

Solution: x = -1 (x = 4 is extraneous)

Section 2.5

21) $\frac{3}{x-3} = \frac{x}{x-3} - \frac{3}{2}$

I will multiply by the LCD of 2(x − 3) to clear the fractions.

$2(x-3)\frac{3}{x-3} = 2(x-3)\frac{x}{x-3} - 2(x-3)\frac{3}{2}$

$2 \cdot 3 = 2x - (x-3)(3)$

$6 = 2x - 3x + 9$

$6 = -x + 9$

$6 - 9 = -x$

$-3 = -x$
$3 = x$

x = 3 is a potential solution, and should be checked.

Check x = 3

$$\frac{3}{3-3} = \frac{3}{3-3} - \frac{3}{2}$$

$$\frac{3}{0} = \frac{3}{0} - \frac{3}{2}$$

I have two fractions with zero in the denominator, both are undefined. I can stop checking. x = 3 does not check and is not a solution. This problem does not have any solutions that check and has no solutions.

Solution: x = 3 (the solution is extraneous)
This problem has no solution.

Chapter 2: Rational Expressions and Equations
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Section 2.5

23) $\frac{5}{x} = \frac{3}{12}$

This is a proportion problem and I can solve by cross multiplying.

5(12) = 3(x)
60 = 3x
20 =x

(this solution will check, but I will not include the checking of the solution for this problem)

Solution: x = 20

27) $\frac{x+1}{2x} = \frac{3}{5}$

This is a proportion problem, and I can solve it by cross multiplying.

5(x+1) = 3(2x)

5x+5 = 6x

5 = 6x – 5x

5 = x

(this solution will check, but I will not include the checking of the solution for this problem)

Solution: x =5

Section 2.6

3) A map has a scale of 20 miles equals 1 inch. How many inches will two cities be apart on the map if they are actually 140 miles apart?

Solve be setting up a proportion.

Let x = number inches cities are apart on the map

$\frac{miles}{inches} \quad \frac{20}{1} = \frac{140}{x}$

(I don't really need to work with the units in my algebra, but I will include units in my answer)

Solve by cross multiplying

$20x = 140$
$x = 7$

Solution: 7 inches

Chapter 2: Rational Expressions and Equations
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Section 2.6

7) A boat travels 10 miles upstream against the current in the same time it takes to go 30 miles downstream with the current. If the speed of the current is 2 miles per hour, find the speed of the boat in still water.

Let b = speed of the boat in still water.

I will create a table to help create the equation I need.

	Distance (d)	Rate (r)	Time (d/r)
With current	30	b+2	$\frac{30}{b+2}$
Against current	10	b-2	$\frac{10}{b-2}$

I can set the time boxes equal to each other, because the boat traveled for the same amount of time in each direction.

$\frac{30}{b+2} = \frac{10}{b-2}$

$10(b+2) = 30(b-2)$

$10b + 20 = 30b - 60$

$80 = 20b$

$4 = \text{b}$

Solution: Boat speed is 4 mph in still water

Chapter 2: Rational Expressions and Equations
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Section 2.6

9) A plane flies 400 miles with the wind in the same time it takes to fly 320 miles against the wind. If the speed of the wind is 20 miles per hour, find the speed of the plane in still air.

Let p = speed of the plane in still air.

I will create a table to help create the equation I need.

	Distance (d)	Rate (r)	Time = d/r
With wind	400	p + 20	$\frac{400}{p+20}$
Against wind	320	p - 20	$\frac{320}{p-20}$

I can set the time boxes equal to each other, because the plane traveled for the same amount of time in each direction.

$\frac{400}{p+20} = \frac{320}{p-20}$

$400(p - 20) = 320(p + 20)$

$400p - 8000 = 320p + 6400$

$80p = 14400$

$p = 180$

Solution: Plane speed 180 mph in still air

Chapter 2: Rational Expressions and Equations
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Section 2.6

11. One person runs 3 miles per hour slower than another. The faster runner can cover 15 miles in the same time the other can run 6 miles. Find the speed of each runner.

Let x = speed of the of the faster runner
Then x – 3 = speed of the slower runner

I will create a table to help create the equation I need.

	Distance (d)	Rate (r)	Time = d/r
Faster runner	15	x	$\frac{15}{x}$
Slower runner	6	x-3	$\frac{6}{x-3}$

I can set the time boxes equal to each other, because the runners ran for the same amount of time.

$\frac{15}{x} = \frac{6}{x-3}$

$15x - 45 = 6x$

$9x = 45$

$x = 5$

(5 is the faster runners speed, subtract 3 from 5 to get the slower runners speed.)

Solution: Faster runs at 5 mph
Slower runs at 2 mph