Chapter 6: Exponential and Logarithmic Functions

Section 6.1: Algebra and Composition of Functions

#1 - 9: Let f(x) = 2x + 3 and g(x) = 3–x. Find each function.

1) (f + g)(x)
2) (g – f)(x)
3) (f/g)(x)
4) $(g \cdot f)(x)$
5) (g/f)(x)
6) $(f \circ g)(x)$
7) $(g \circ f)(x)$
8) (g+f)(x)
9) (f – g)(x)

#10 - 18: Let f(x) = $2x^2$ – 5x – 3 and g(x) = 5x+4. Find each function.

10) (f + g)(x)
11) (g – f)(x)
12) (f/g)(x)
13) $(g \cdot f)(x)$
14) (g/f)(x)
15) $(f \circ g)(x)$
16) $(g \circ f)(x)$
17) (g+f)(x)
18) (f – g)(x)

#19 - 30: Let h(x) = 2x + 3 and k(x) = 3 – x. Find each of the following.

19) (h+k)(3)
20) (hk)(-1)
21) (h/k)(5)
22) (k– h)(0)
23) (h– k)(7)
24) (kh)(4)
25) $(h \circ k)(4)$
26) $(h \circ k)(0)$
27) $(k \circ h)(3)$
28) $(h \circ k)(-2)$
29) $(k \circ h)(1)$
30) $(k \circ h)(-6)$

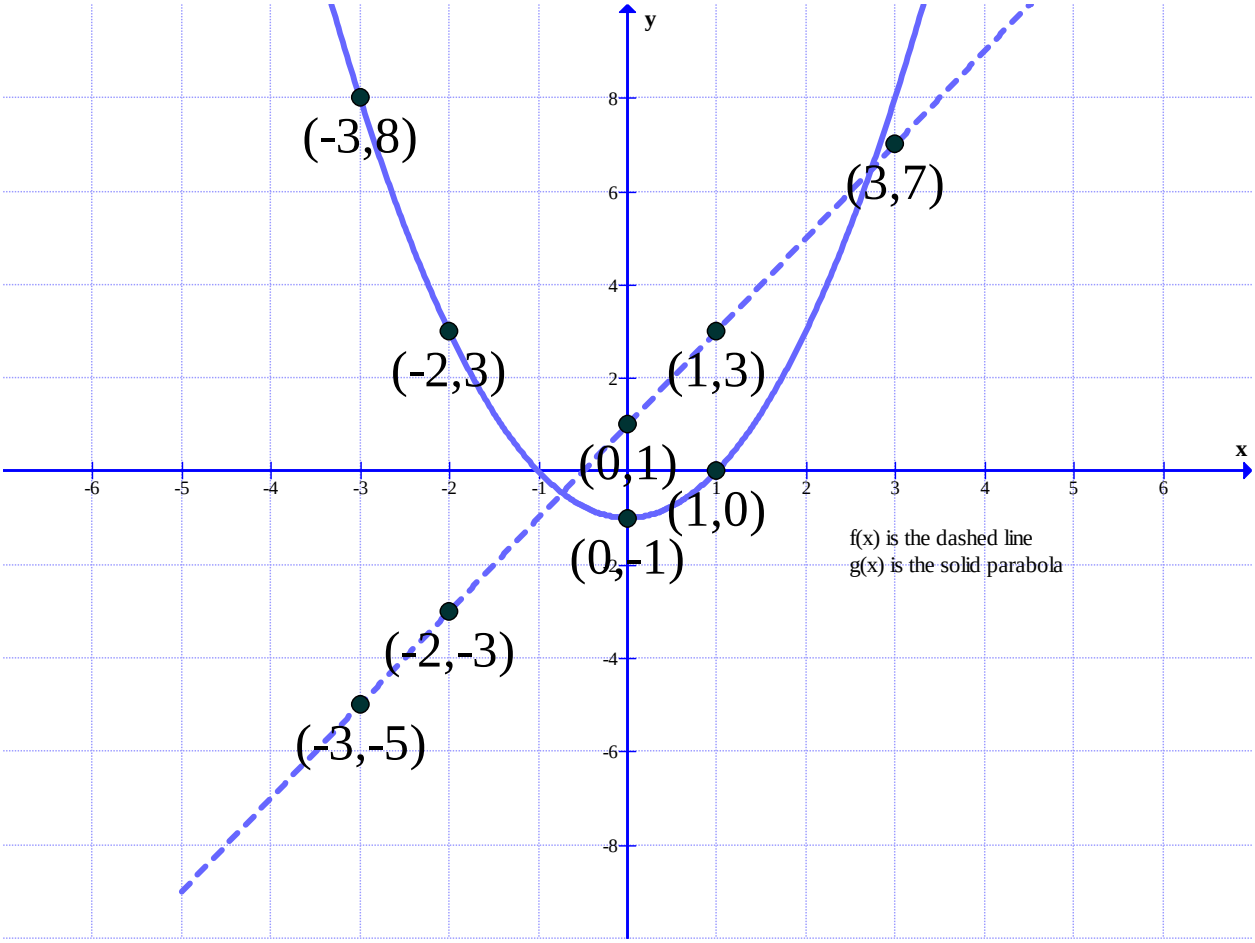
#31 - 39: Let s(x) = $2x^2$ – 5x – 3 and t(x) = 5x+4. Find each of the following.

31) (s/t)(3)
32) (s– t)(4)
33) (t+s)(6)
34) $(s \circ t)(4)$
35) $(s \circ t)(0)$
36) $(s \circ t)(3)$
37) $(s \circ t)(-2)$
38) $(t \circ s)(1)$
39) $(t \circ s)(-6)$

Chapter 6: Exponential and Logarithmic Functions

Section 6.1: Algebra and Composition of Functions

#40 - 48: Refer to the graph and find the following.



40) (f+g)(-2)

41) (fg)(0)

42) (g– f)(-3)

43) (g/f)(1)

44) (f/g)(0)

45) (gf)(-2)

46) $(f \circ g)(1)$

47) $(g \circ f)(-2)$

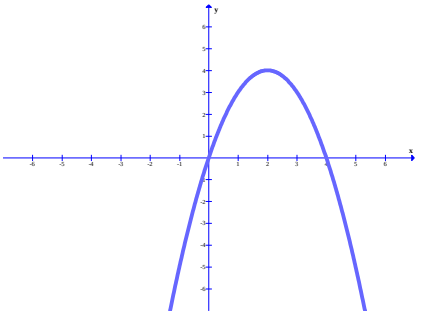
48) $(f \circ g)(-2)$

Chapter 6: Exponential and Logarithmic Functions

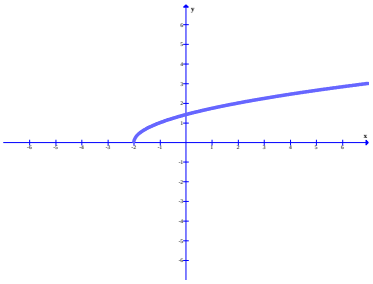
Section 6.2: Inverse Functions

#1 - 4: Determine if the functions are one to one by using the horizontal line test.

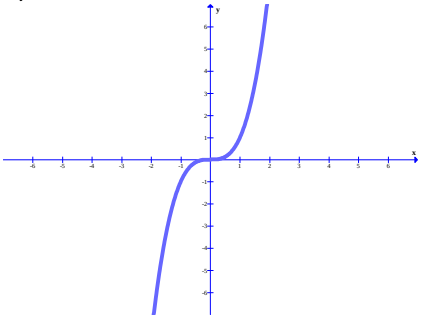
1)



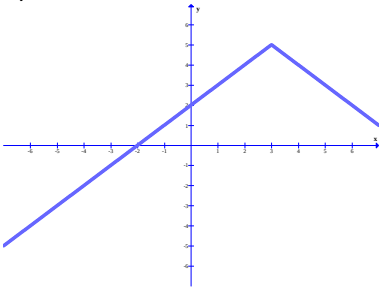
2)



3)



4)



#5 - 12: Sketch a graph and determine whether each function is one to one (you may construct a table of values, or use a technique you already have learned to construct your graph.)

5) $f(x) = 2x - 5$

6) $g(x) = x + 3$

7) $f(x) = x^2 - 3$

8) $k(x) = (x - 3)^2 + 6$

9) $g(x) = x^4$ (use -2,-1,0,1,2 table)

10) $k(x) = x^4 - 3$ (use -2,-1,0,1,2 table)

11) $f(x) = x^3$ (use -2,-1,0,1,2 table)

12) $h(x) = x^3 - 2$ (use -2,-1,0,1,2 table)

#13 - 18: Determine which of the functions are one to one. If a function is one to one find its inverse.

13) f = { (0,1) (1,4) (2,4) (3,5)}

14) g = { (3,2) (4,5) (-3,4) (1,5) (0,6)}

15) h = {(0,3) (5,1) (7,11) (9, -3)}

16) k = { (-3,4) (-5,6) (9, -3) (4, 0)}

17) m = { (0,2) (2,3) (3, 5)}

18) n = { (1,1) (2,2) (3,4) (5,5)}

Chapter 6: Exponential and Logarithmic Functions

Section 6.2: Inverse Functions

#19 - 28: Each of the following functions is one to one. Find the inverse of each function, and express it using appropriate notation.

19) f(x) = 2x − 4

20) f(x) = 3x − 6

21) $g(x) = \frac{x-2}{3}$

22) $g(x) = \frac{2x-5}{7}$

23) $h(x) = \frac{2}{x}$

24) $h(x) = \frac{3}{x}$

25) $m(x) = \sqrt[3]{x}$

26) $m(x) = \sqrt[3]{x-2}$

27) $f(x) = x^3 + 2$

28) $f(x) = x^3 - 3$

Chapter 6: Exponential and Logarithmic Functions

Section 6.3: Exponential Functions

#1 - 6: Evaluate the expression, use a calculator when needed. Do not write answers in decimal form.

1) $49^{3/2}$ 2) 5^{-3} 3) $36^{-1/2}$ 4) 8^{-2} 5) $125^{-1/3}$ 6) $64^{2/3}$

#7 - 12: Evaluate the expression with the help of a calculator. Round your answer to 3 decimal places.

7) $2^{1.25}$ 8) $3^{2.26}$ 9) $5^{-2.6}$ 10) $5^{\sqrt{2}}$ 11) $3^{\sqrt[3]{4}}$ 12) $2^{-\sqrt{3}}$

#13 - 22: Plot 5 points and sketch a graph of the function.

13) $f(x) = 3^x$ 14) $g(x) = 2^x$ 15) $h(x) = \left(\frac{1}{3}\right)^x$

16) $k(x) = \left(\frac{1}{2}\right)^x$ 17) $f(x) = 2^{x-3}$ 18) $m(x) = 3^{x-4}$

19) $f(x) = 2^{x+4}$ 21) $g(x) = 3^{x+1}$ 22) $f(x) = \left(\frac{1}{2}\right)^{x-1}$

23) The number of computers infected by the spread of a virus through email can be described by the exponential function $c(t)=4(1.02)^t$, where t is the number of minutes since the first infected e-mail was opened. Approximate the number of computers that will be infected after 6 hours (240 minutes) (round to the nearest whole number).

24) A colony of 2 million bacteria is growing in a culture medium. The population P of bacteria after t hours is given by the function $P(t)=2{,}000{,}000(2.3)^t$, Find the population of the culture in 12 hours. (round to the nearest million)

25) The charge remaining in a battery decreases as the battery discharges. The charge C (in coulombs) after t days is given by the formula $C(t)= 0.0003(0.7)^t$. Find the charge after 5 days. (round your answer to 5 decimal places)

26) Five hundred grams of a radioactive material decays according to the formula $A = 500\left(\frac{2}{3}\right)^t$, where t is measured in years. Find the amount present in 10 years. (Round to the nearest one-tenth of a gram.)

Chapter 6: Exponential and Logarithmic Functions

Section 6.3: Exponential Functions

#27-30: Use the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to answer the following.

27) An initial deposit of $1,000 earns 4% interest compounded twice per year. How much will be in the account after 5 years?

28) An initial deposit of $1,000 earns 3% interest compounded monthly. How much will be in the account after 10 years?

29) An initial deposit of $10,000 earns 6% interest compounded quarterly. How much will be in the account after 7 years?

30) An initial deposit of $20,000 earns 3% interest compounded quarterly. How much will be in the account after 17 years?

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Chapter 6: Exponential and Logarithmic Functions

Section 6.4: Logarithmic Functions

#1 - 9: Write the equation in logarithmic form.

1) $3^2 = 9$ 2) $2^3 = 8$ 3) $3^4 = 81$

4) $2^5 = 32$ 5) $3^{-1} = \frac{1}{3}$ 6) $4^{-2} = \frac{1}{16}$

7) $e^y = x$ 8) $4^w = c$ 9) $m^3 = z$

#10 - 18: Write the equation in exponential form.

10) $\log_2 8 = 3$ 11) $\log_3 81 = 4$ 12) $\log_2 16 = 4$

13) $\log_2 64 = 6$ 14) $\log_5 1 = 0$ 15) $\log_6 6 = 1$

16) $\log_3 x = 2$ 17) $\log x = 3$ 18) $\log x = 4$

#19 - 42: Find the logarithm value without using a calculator.

19) $\log_2 2$ 20) $\log_2 1$ 21) $\log_7 7$

22) $\log_3 3$ 23) $\log_3 1$ 24) $\log_3 9$

25) $\log_4 1$ 26) $\log_4 4$ 27) $\log_4 64$

28) $\log_5 1$ 29) $\log_5 5$ 30) $\log_5 125$

31) $\log 1$ 32) $\log 10$ 33) $\log 100$

34) $\log_2 32$ 35) $\log_2 128$ 36) $\log_3 243$

37) $\log_2 2^3$ 38) $\log_3 3^4$ 39) $\log_5 5^6$

40) $\log_7 7^8$ 41) $\log_4 4^5$ 42) $\log_2 2^9$

#43 - 48: Use a calculator to approximate the logarithms. Round to 4 decimal places.

43) $\log 6$ 44) $\log 5$ 45) $log\frac{1}{3}$

46) $log\left(\frac{1}{10}\right)$ 47) $\log(10^{-3})$ 48) $log\left(\frac{1}{100}\right)$

#49 - 57: Graph the logarithmic functions. First write the equation in exponential form, then create a table of values and plot the points. State the domain of each function.

49) $y = \log_2 x$ 50) $y = \log_3 x$ 51) $y = \log_2(x - 3)$

52) $y = \log_3(x - 2)$ 53) $y = \log_2(x+4)$ 54) $y = \log_3(x+1)$

55) $y = log_{1/2}x$ 56) $y = log_{1/3}x$ 57) $y = log_{1/2}(x + 1)$

Chapter 6: Exponential and Logarithmic Functions

Section 6.5: Base-e, Exponential Functions

#1 - 12: Use your calculator to evaluate, round to 2 decimal places.

1) e^2 2) e^3 3) e^{-1}

4) e^{-2} 5) e^{π} 6) ln(1)

7) ln(e) 8) $\ln(e^2)$ 9) $\ln(e^3)$

10) ln(4) 11) ln(0.25) 12) ln(0.3)

#13 - 18: Make a table of values and graph the function. Round to 2 decimal places.

13) $y= e^x$ 14) $y=e^{x+3}$ 15) $y=e^{x+3}$

16) $y= e^{x+2}$ 17) $y = e^{x-2}$ 18) $y = e^{x-3}$

#19 - 27: Graph the logarithmic functions. First write the equation in exponential form, then create a table of values and plot the points (round to 2 decimal places). State the domain of each function.

19) y = ln(x) 20) y=ln(x – 1) 21) y = ln(x – 2)

22) y = ln(x+3) 23) y = ln(x+2) 24) y = ln(-x)

25) y = ln(3– x) 26) y = ln(2– x) 27) y = ln(2x)

#28-31: Use the formula $A=Pe^{rt}$ to answer the following.

28) An initial investment of $5,000 earns 6% interest compounded continuously. What will the investment be worth in 5 years?

29) An initial investment of $10,000 earns 5.25% interest compounded continuously. What will the investment be worth in 8 years?

30) An initial investment of $25,000 earns 4% interest compounded continuously. What will the investment be worth in 11 years?

31) An initial investment of $500 earns 2.25% interest compounded continuously. What will the investment be worth in 8 years?

Chapter 6: Exponential and Logarithmic Functions

Section 6.6: Properties of Logarithms

#1 - 12: Evaluate each expression. Check your answer using your calculator.

1) $\log_2 16$ 2) $\log_3 9^2$ 3) $\log_4 16^3$

4) $\log_7 1$ 5) $\log_8 8^5$ 6) $\log_6 216$

7) $\log_3 243$ 8) $\ln(e^4)$ 9) $\ln(e)$

10) $\ln(1)$ 11) $\log_2 64^3$ 12) $\log_5 25^2$

13) Which of these is a true statement?
a) $\log_2(4*8) = \log_2 4 * \log_2 8$
b) $\log_2(4*8) = \log_2 4 + \log_2 8$

14) Which of these is a true statement?
a) $\log_3(3*27) = \log_3 3 + \log_3 27$
b) $\log_3(3*27) = \log_3 3 * \log_3 27$

15) Which of these is a true statement?
a) $log_2 \frac{16}{2} = \frac{log_2 16}{log_2 2}$
b) $log_2 \frac{16}{2} = log_2 16 - log_2 2$

16) Which of these is a true statement?
a) $log_3 \frac{81}{3} = \frac{log_3 81}{log_3 3}$
b) $log_3 \frac{81}{3} = log_3 81 - log_3 3$

17) Which of these is a true statement?
a) $\log 100^3 = (\log 100)^3$
b) $\log 100^3 = = 3\log 100$

18) Which of these is a true statement?
a) $\log 100^4 = 4\log 100$
b) $\log 100^4 = (\log 100)^4$

#19 – 30: Expand into sums and differences of logarithms

19) $\log_3 (x^2y^3)$ 20) $\log_4(xy^3z^5)$ 21) $\log_5(25x^2y^6)$

22) $log_b \frac{x^4}{y}$ 23) $log_2 \frac{xy^3}{z^2}$ 24) $log_7 \frac{x}{yz^3}$

25) $log_2 \frac{xy}{w^2z^5}$ 26) $\log_3 z^3w^2$ 27) $\log_4 x^{-3}y$

28) $log_4(y\sqrt{z})$ 29) $log_2(x^2 \cdot \sqrt[3]{y})$ 30) $log_5 \frac{\sqrt{x}}{\sqrt[3]{y}}$

#31 - 42: Write the expression as a single logarithm.

31) $3\log_2 x + 4\log_2 y$ 32) $5\ln x + \ln y + 3\ln z$ 33) $2\log_3 x + 4\log_3 y + \log_3 z$

34) $2\log x - 3\log y$ 35) $5\log_2 x + 3\log_2 y - \log_2 z$ 36) $4\log x + 2\log y - 3\log z$

37) $4\log x - 2\log y - 3\log z$ 38) $2\log_5 x - 4\log_5 y + \log_5 z$ 39) $-2\log_3 x + \log_3 y + \log_3 z$

40) $-\ln x + \ln y + \ln z$ 41) $\ln x + 3\ln x - 2\ln z$ 42) $4\ln x - 2\ln x + 5\ln x$

Chapter 6: Exponential and Logarithmic Functions

Section 6.6: Properties of Logarithms

#43 - 51: Evaluate using a calculator, round your answer to 2 decimal places.

43) $\log_2 3$ 44) $\log_7 14$ 45) $\log_3 5$

46) $\log_9 36$ 47) $\log_4 0.65$ 48) $\log_2 0.25$

49) $\log_5 18$ 50) $\log_6 7$ 51) $\log_9 0.123$

Chapter 6: Exponential and Logarithmic Functions

Section 6.7: Exponential and Logarithmic Equations

#1 - 12: Solve the exponential equation by writing each side of the equation with the same base then equating the exponents.

1) $2^x = 16$

2) $3^x = 27$

3) $2^{x+1} = 32$

4) $3^{x+2} = 81$

5) $\left(\frac{1}{2}\right)^x = 16$

6) $\left(\frac{1}{3}\right)^x = 27$

7) $2^{4-x} = 64$

8) $3^{5-x} = 243$

9) $32^x = 2$

10) $27^x = 3$

11) $16^x = 4$

12) $49^x = 7$

#13 - 24: Use logarithms to solve. (round to two decimals)

13) $3^x = 6$

14) $2^x = 10$

15) $e^x = 12$

16) $e^x = 1.15$

17) $5(10^x) = 20$

18) $4(3^x)=24$

19) $32e^{2x} = 128$

20) $14e^{3x} = 42$

21) $2e^{.06x} = 6$

22) $3e^{.04x} = 12$

23) $4e^{.025x} = 8$

24) $3e^{.05x} = 21$

#25 - 51: Solve the logarithmic equations, round to 2 decimals when needed.

25) $\log_3 x = 2$

26) $\log_2 x = 3$

27) $\ln x = 1$

28) $\ln x = 0$

29) $\log_3(x-2) = 2$

30) $\log_4(5x-6) = -2$

31) $\log_4(3x-6) = -1$

32) $\log_2(x-3) = -4$

33) $\log_2(x-1) = -3$

34) $\log_2(x-1) = 5$

35) $\log_2(x-1) = 3$

36) $\log_3(x-5)=2$

37) $\log_2(2x)=5$

38) $\log_3(3x)=0$

39) $\log(x+1) = \log(3x-2)$

40) $\ln(x-4) = \ln(2x-10)$

41) $\log_2(x+3) = \log_2(3x)$

42) $\log_4(3x+6)=\log_4(4x)$

43) $\log_2 x - \log_2(x+6)=-2$

44) $\log_3 x - \log_3(x+6) = -1$

45) $\log_2 x - \log_2(x-6) = 2$

46) $\log_3(x-5) - \log_3(x+3) = -2$

47) $\log_2(x+6) - \log_2(3x+2)=-1$

48) $\log_2(x+2)+\log_2 x = 3$

49) $\log_3 x + \log_3(x+6) = 3$

50) $\log_4 x + \log_4(x+12) = 3$

51) $\log_3(x+6) + \log_3(3x) = 4$

Chapter 6: Exponential and Logarithmic Functions

Section 6.7: Exponential and Logarithmic Equations

I deleted #52 - 55

#56-59: Use the formula $P = P_0e^{kt}$, where P_0 is the initial population at t=0, and k is the rate of growth.

56) The rodent population in a given city rose from 20,000 to 30,000 in 1 year. How long will it take the population to reach 100,000? (round to the nearest year)

57) The bacterial in a laboratory culture increased from an initial population of 500 to 1,500 in 3 hours. How long will it take the population to reach 10,000? (hint first use the 500 to 1,500 in 3 hours and the formula above to solve for k, then use the 10,000 to answer the question.) (round to the nearest hour)

58) The population of a city is expected to triple in 20 years. The city currently has 10,000 residents. How long will it take to get to 50,000 residents? (round to the nearest year)

59) The population of a city is expected to double in 20 years. The city currently has 1,000 residents. How long will it take to get to 5,000 residents? (round to the nearest year)

#60-63, Use the continuous compound interest formula $A = Pe^{rt}$, where P is the initial investment, r the annual interest rate, and t the number of years to solve the following.

60) How long will it take an initial investment of $1,000 to triple if it is expected to earn 6% interest compounded continuously? (Round to 1 decimal place)

61) How long will it take an initial investment of $1,000 to double if it is expected to earn 6% interest compounded continuously? (Round to 1 decimal place)

62) How long will it take an initial investment of $100,000 to grow to $1,000,000 if it is expected to earn 4% interest compounded continuously? (Round to 1 decimal place)

63) How long will it take an initial investment of $10,000 to grow to $15,000 if it is expected to earn 4% interest compounded continuously? (Round to 1 decimal place)

Chapter 6: Exponential and Logarithmic Functions

Chapter 6: Review

#1 - 3: Let $f(x) = x^2 - 3x$ and $g(x) = 2x - 6$. Find each function.

1) (f + g)(x) 2) (f/g)(x) 3) $(f \circ g)(x)$

#4 - 6: Let $h(x) = x^2 - 9$ and $k(x) = 3 - x^2$. Find each of the following.

4) (h+k)(2) 5) (h/k)(-1) 6) $(k \circ h)(0)$

#7 - 8: Determine which of the functions are one to one. If a function is one to one find its inverse.

7) f = { (0,-1) (1,-4) (2,4) (3,5)} 8) g = { (1,2) (4,16) (-3,4) (7,5) (0,2)}

#9 - 10: Each of the following functions is one to one. Find the inverse of each function, and express it using appropriate notation.

9) f(x) = 6x+8 10) $h(x) = \frac{2x+3}{4}$

#11-12: Plot 5 points and sketch a graph of the function.

11) $f(x) = 3^{x+5}$ 12) $h(x) = \left(\frac{1}{3}\right)^{x-4}$

13) Use the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to answer the following. An initial deposit of $10,000 earns 6% interest compounded monthly. How much will be in the account after 25 years?
answer $44649.70

14) Write the equation in logarithmic form. $e^2 = y$

#15 - 17: Find the logarithm value without using a calculator.

15) $\log_2 16$ 16) $\log_9 1$ 17) $\log_2 2^{10}$

18) Graph the logarithmic function. First write the equation in exponential form, then create a table of values and plot the points. State the domain of each function. $y = \log_2(x - 3)$

#19 - 20: Expand into sums and differences of logarithms.

19) $log_2 \frac{x^3}{y^4 z^2}$ 20) $\log_4(xy^3z^5)$

#21 - 22: Write the expression as a single logarithm.

21) $3\log_2 x - 4\log_2 y$ 22) $5\ln x - \ln y - 3\ln z$

Chapter 6: Exponential and Logarithmic Functions

Chapter 6: Review

#23 - 24: Solve the exponential equation by writing each side of the equation with the same base then equating the exponents.

23) $2^{x-5} = 16$

24) $\left(\frac{1}{2}\right)^{x+1} = 16$

#25 - 26: Use Logs to solve. (round to two decimals)

25) $3^{-x} = 6$

26) $18\,e^{x} = 12$

#27 - 31: Solve the logarithmic equations, round to 2 decimals when needed.

27) $\log_2 x = -1$

28) $\log_2(x - 1) = 5$

29) $\log(x - 1) = \log(3x - 5)$

30) $\log_3(x+6) - \log_3(x) = 1$

31) How long will it take an initial investment of $1,000 to triple if it is expected to earn 6% interest compounded continuously? (Round to 1 decimal place) **answer 18.3 years**