Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.1

1) (f + g)(x)
= f(x) + g(x) = (2x+3) + (3-x) = 2x+3 + 3 – x = x + 6

The function is defined for all values of x, so the domain is all real numbers.

Solution: (f + g)(x)**= x + 6**

3) (f/g)(x)

$= \frac{f(x)}{g(x)} = \frac{2x+3}{3-x}$

To find the domain, ignore the numerator.
Find where the denominator equals zero.
Exclude this number.
domain : computation 3 – x = 0
3 = x

Solution: (f/g)(x) = $\frac{2x+3}{3-x}$

7) $(g \circ f)(x)$

$= g(f(x)) = g(2x+3)$
$= 3-(2x+3) = 3-2x-3 = -2x$

The function is defined for all values of x, so the domain is all real numbers.

Solution: $(g \circ f)(x)$= -2x

11) (g – f)(x)

$= g(x) - f(x) = (5x \; + 4) - \; (2x^2 - 5x - 3)$
$= 5x + 4 - 2x^2 + 5x + 3 = -2x^2 + 10x + 7$

The function is defined for all values of x, so the domain is all real numbers.

Solution: (g – f)(x) = $-2x^2 + 10x + 7$

13) $(g \cdot f)(x)$

$= g(x) * f(x) = (5x+4)(2x^2 - 5x - 3)$
$= \; 10x^3 - 25x^2 - 15x + 8x^2 - 20x - 12$
$=10x^3 - 17x^2 - 35x - 12$

The function is defined for all values of x, so the domain is all real numbers.

Solution: $(g \cdot f)(x)$ = $10x^3 - 17x^2 - 35x - 12$

15) $(f \circ g)(x)$
$= f(g(x)) = f(5x+4)$
$= 2(5x+4)^2 - 5(5x+4) - 3$
$= 2(5x+4)(5x+4) - 25x - 20 - 3$
$=2(25x^2 + 20x + 20x + 16) - 25x - 20 - 3$
$= 50x^2 + 40x + 40x + 32 - 25x - 20 - 3$
$=50x^2 + 55x + 9$

The function is defined for all values of x, so the domain is all real numbers.

Solution: $(f \circ g)(x)$= $50x^2 + 55x + 9$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems
Section 6.1

19) (h+k)(3)
$= h(3) + k(3)$

$= [2(3) + 3] + (3 - 3) = 9 + 0 = 9$

Solution: (h+k)(3)=9

21) (h/k)(5)

$= \frac{h(5)}{k(5)} = \frac{2(5)+3}{3-5} = \frac{13}{-2} = -\frac{13}{2}$

Solution: (h/k)(5) = $-\frac{13}{2}$

25) $(h \circ k)(4)$

$= h(k(4)) = h(3-4) = h(-1)$

$= 2(-1) + 3 = -2 + 3 = 1$

Solution: $(h \circ k)(4)$ = 1

27) $(k \circ h)(3)$

$= k(h(3)) = k(2 * 3 + 3)$

$= k(9) = 3 - 9 = -6$

Solution: $(k \circ h)(3)$= -6

31) $\left(\frac{s}{t}\right)(3)$

$= \frac{s(3)}{t(3)} = \frac{2(3)^2 - 5(3) - 3}{5*3+4}$

$= \frac{18-15-3}{15+4} = \frac{0}{19} = 0$

Solution: $\left(\frac{s}{t}\right)(3) = 0$

35) $(s \circ t)(0)$

$= s(t(0)) = s(5 * 0 + 4) = s(4)$

$= 2(4)^2 - 5 * 4 - 3 = 32 - 20 - 3 = 9$

Solution: $(s \circ t)(0)$ = 9

37) $(s \circ t)(-2)$

$= s(t(-2)) = s(5(-2) + 4)$

$= s(-6) = 2(-6)^2 - 5(-6) - 3$

$= 72 + 30 - 3 = 99$

Solution: $(s \circ t)(-2)$= 99

41) (fg)(0)
$= f(0) * g(0) = 1 * (-1) = -1$

To find f(0) look at the graph.
This is asking for the y coordinate of the point that has 0 for its x coordinate (f(0) = 1)

To find g(0) look at the graph.
This is asking for the y coordinate of the point that has 0 for its x coordinate. (g(0) = -1)

Solution: (fg)(0)=-1

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems
Section 6.1

43) (g/f)(1)

$= \frac{g(1)}{f(1)} = \frac{0}{3} = 0$

To find f(1) look at the graph.
This is asking for the y coordinate of the point that has 0 for its x coordinate. (f(1) = 3)

To find g(1) look at the graph.
This is asking for the y coordinate of the point that has 0 for its x coordinate. (g(1) = 0)

Solution: (g/f)(1)= 0

47) $(g \circ f)(-2)$
First write without composite symbol.

=g(f(-2)) ****

Then find f(-2), which is the y coordinate of the point on the f graph with x of -2.

f(-2)=-3

replace f(-2) with 3 in **** above.

=g(-3)

Now find g(-3) which is the y coordinate of the point on the g graph with x = -3. g(-3) =8

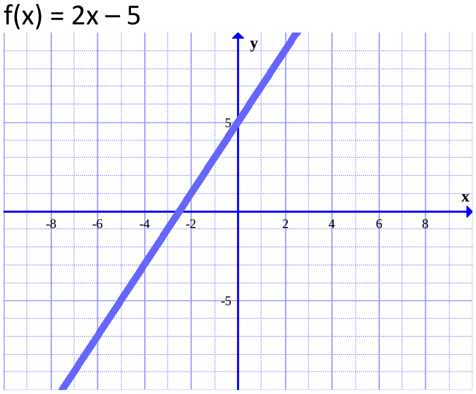
Solution: $(g \circ f)(-2)$=8

Section 6.2

#5-12, Sketch a graph and determine whether each function is one to one (you may construct a table of values, use a graphing calculator, or use a technique you already have learned to construct your graph.)

5) f(x) = 2x – 5

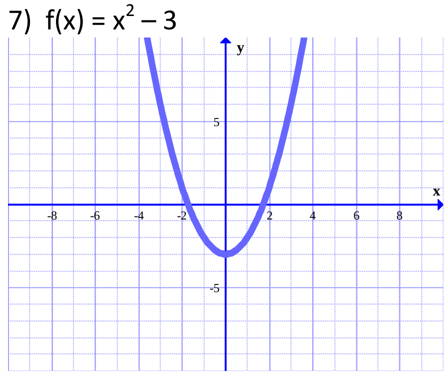
f(x) = 2x – 5



Function is one-to-one

no horizontal line can be drawn to touch the graph in more than one place, the function is one-to-one.

7) f(x) = x^2 – 3

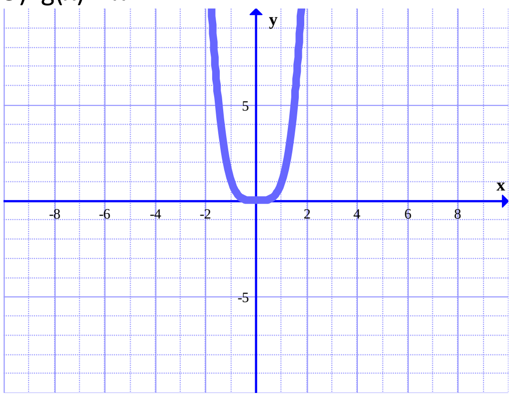


Function is not one-to-one

A horizontal line can be drawn to touch the graph in more than one place, the function not is one-to-one.

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems
Section 6.2

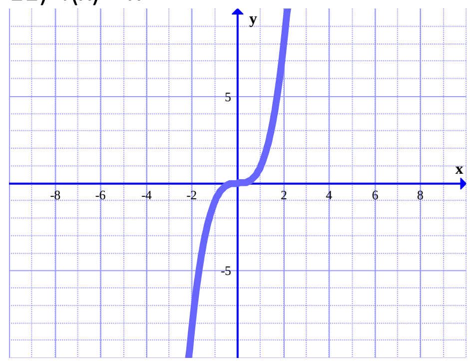
9) $g(x) = x^4$



Solution: Function is not one-to-one

A horizontal line can be drawn to touch the graph in more than one place, the function not is one-to-one.

11) $f(x) = x^3$



Solution: Function is one-to-one

No horizontal line can be drawn to touch the graph in more than one place, the function is one-to-one.

13) f = { (0,1) (1,4) (2,4) (3,5)}

The function is not one-to-one because two different points have the same y-value.

Solution: Function is not one-to-one

15) h = {(0,3) (5,1) (7,11) (9, -3)}

Function is one-to-one because all of the y values are different. Find inverse by switching the x and y's.

Solution: Function is one-to-one
h^{-1} = { (3,0) (1,5) (11,7) (-3,9)}

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems
Section 6.2

19) f(x) = 2x – 4
first: replace function symbol with a y
y = 2x - 4
second: switch x and y, this creates the inverse
x = 2y - 4
third: solve for y
$x + 4 = 2y$
$\frac{x+4}{2} = y$
fourth: write with y on left side (if not already there)
$y = \frac{x+4}{2}$
fifth: replace the y with an inverse symbol

Solution: $\boldsymbol{f^{-1}(x) = \frac{x+4}{2}}$

21) $g(x) = \frac{x-2}{3}$
first: replace function symbol with a y
$y = \frac{x-2}{3}$
second: switch x and y, this creates the inverse
$x = \frac{y-2}{3}$
third: solve for y
$3x = 3 * \frac{y-2}{3}$
3x = y – 2
3x+ 2 = y
fourth: write with y on left side (if not already there)
y = 3x + 2
fifth: replace the y with an inverse symbol
Solution: $\boldsymbol{g^{-1}(x) = 3x + 2}$

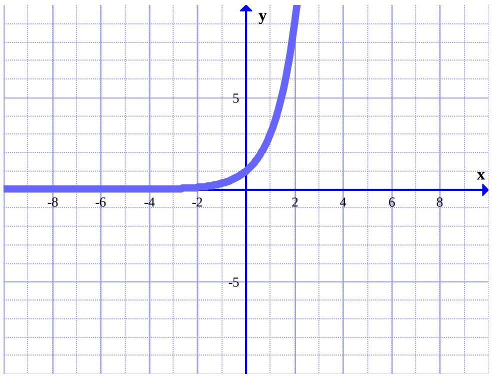
25) $m(x) = \sqrt[3]{x}$
first: replace function symbol with a y
$y = \sqrt[3]{x}$
second: switch x and y, this creates the inverse
$x = \sqrt[3]{y}$
third: solve for y
$x^3 = \left(\sqrt[3]{y}\right)^3$
$x^3 = y$
fourth: write with y on left side (if not already there)
$y = x^3$
fifth: replace the y with an inverse symbol
Solution: $\boldsymbol{m^{-1}(x) = x^3}$

27) f(x) = x^3 + 2
first: replace function symbol with a y
y = x^3 + 2
second: switch x and y, this creates the inverse
x=y^3+2
third: solve for y
x-2 = y^3
$\sqrt[3]{x-2} = \sqrt[3]{y^3}$
$\sqrt[3]{x-2} = y$
fourth: write with y on left side (if not already there)
$y = \sqrt[3]{x-2}$
fifth: replace the y with an inverse symbol
Solution: $\boldsymbol{f^{-1}(x) = \sqrt[3]{x-2}}$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

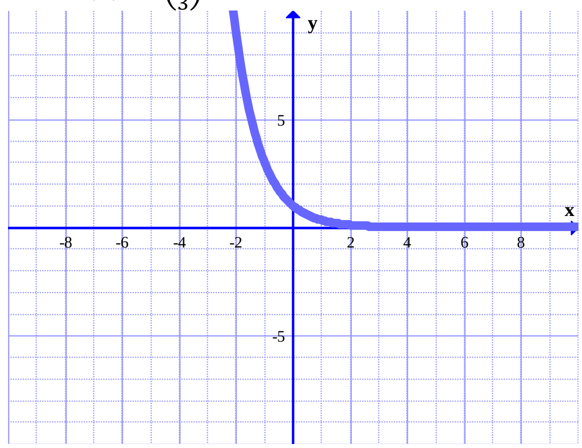
Section 6.3

13) $f(x) = 3^x$



x	f(x)	computations
0	1	$f(0)=3^0 = 1$
1	3	$f(1)=3^1 = 3$
-1	1/3	$f(-1) = 3^{-1} = 1/3$
-2	1/9	$f(-2)=3^{-2} = 1/3^2 = 1/9$

15) $h(x) = \left(\frac{1}{3}\right)^x$

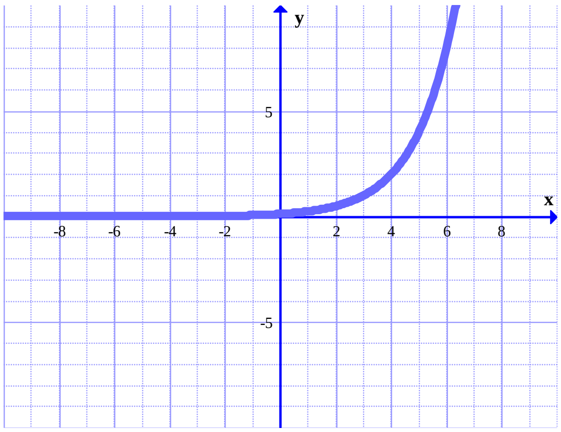


x	h(x)	computations
0	1	$h(0) = \left(\frac{1}{3}\right)^0 = 1$
-1	1/3	$h(1) = \left(\frac{1}{3}\right)^1 = \frac{1}{3}$
1	3	$h(-1) = \left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1 = 3$
-2	9	$h(-2) = \left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^2 = 9$
2	1/9	$h(2) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$
-3	27	$h(-3) = \left(\frac{1}{3}\right)^{-3} = \left(\frac{3}{1}\right)^3 = 27$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

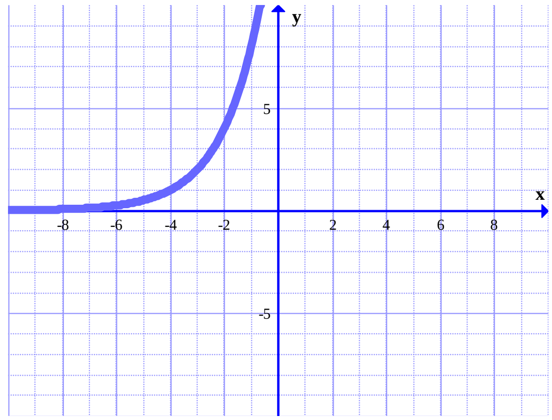
Section 6.3

17) $f(x) = 2^{x-3}$



x	f(x)	computations
3	1	$f(3) = 2^{3-3} = 2^0 = 1$
4	2	$f(4) = 2^{4-3} = 2^1 = 2$
2	½	$f(2)=2^{2-3} = 2^{-1} = 1/2$
5	4	$f(5)= 2^{5-3} = 2^2 = 4$
1	1/4	$f(1)=2^{1-3} = 2^{-2} = ½^2 = 1/4$

19) $f(x) = 2^{x+4}$



x	f(x)	computations
-4	1	$f(-4)=2^{-4+4} = 2^0 = 1$
-5	½	$f(-5) = 2^{-5+4} = 2^{-1} = 1/2$
-3	2	$f(-3) = 2^{-3+4} = 2^1 = 2$
-6	1/4	$f(-6) = 2^{-6+4} = 2^{-2} = ½^2 = 1/4$

23) The number of computers infected by the spread of a virus through email can be described by the exponential function $c(t)=4(1.02)^t$, where t is the number of minutes since the first infected e-mail was opened. Approximate the number of computers that will be infected after 6 hours (240 minutes). (round to the nearest computer)

I just need to plug in 240 for t in the equation. I will need to use my calculator to get this answer.

$c(240) = 4(1.02)^{240} = 464$

Solution: 464 computers

25) The charge remaining in a battery decreases as the battery discharges. The charge C (in coulombs) after t days is given by the formula $C(t)= 0.0003(0.7)^t$. Find the charge after 5 days. (carry 6 decimal places in your answer)

I need to evaluate the function at t = 5. I will need my calculator to simplify.

$C(0) = 0.0003(0.7)^5 = 0.00005$

Solution: 0.00005 coulombs

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.3

#26-30: Use the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to answer the following.

27) An initial deposit of $1,000 earns 4% interest compounded twice per year. How much will be in the account after 5 years?

P = 1,000
r = .04
n = 2
t = 5

Substitute these values in the formula. Then use my calculator to evaluate.

$1000\left(1 + \frac{.04}{2}\right)^{2*5}$=1218.99

Solution: $1218.99

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.4

1) $3^2 = 9$

The base of the exponential function (3) is the base or subscripted part of the logarithm. Switch the 2 and the 9.

Solution: $\log_3 9 = 2$

5) $3^{-1} = \frac{1}{3}$

The base of the exponential function (3) is the base or subscripted part of the logarithm. Switch the -1 and the 1/3.

Solution: $\log_3(1/3) = -1$

7) $e^y = x$

The base of the exponential function (e) is the base or subscripted part of the logarithm. Switch the y and the x.

Solution: $\log_e x = y$

11) $\log_3 81 = 4$

Write the problem without the log.
The 3 remains the base in the exponential function. Switch the 81 and 4.

Solution: $3^4 = 81$

13) $\log_2 64 = 6$

Write the problem without the log.
The 2 remains the base in the exponential function. Switch the 64 and 6.

Solution: $2^6 = 64$

17) $\log x = 3$

Since no base is written the base is assumed to be 10. You can think of the problem like this:

$\log_{10} x = 3$

Write the problem without the log.
The 10 remains the base in the exponential function. Switch the x and 3.

Solution: $10^3 = x$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.4

19) $\log_2 2$

This is asking the question $2^? = 2$,
The answer must be 1 because: $2^1 = 2$

Solution: 1

23) $\log_3 1$

This is asking the question $3^? = 1$,
The answer must be 0 because: $3^0 = 1$

Solution: 0

27) $\log_4 64$

This is asking the question $4^? = 64$,
The answer must be 3 because: $4^3 = 64$

Solution: 3

31) $\log 1$

Remember if no base is written the base is assumed to be a 10. Think of the problem as:

$\log_{10} 1$

This is asking the question $10^? = 1$,
The answer must be 0 because: $10^0 = 1$

Solution: 0

33) $\log 100$

This is asking the question $10^? = 100$,
The answer must be 2 because: $10^2 = 100$

Solution: 2

37) $\log_2 2^3$

This is asking the question $2^? = 2^3$,
The answer must be 3 because: $2^3 = 2^3$

Solution: 3

41) $\log_4 4^5$

This is asking the question $4^? = 4^5$,
The answer must be 5 because: $4^5 = 4^5$

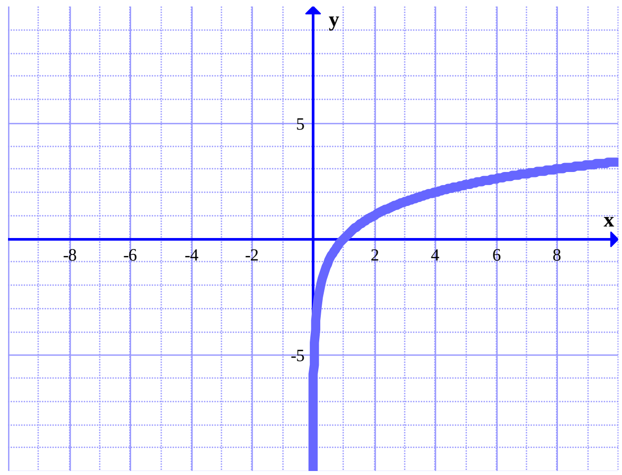
Solution: 5

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.4

49) $y = \log_2 x$

$2^y = x$ (problem written in exponential form)

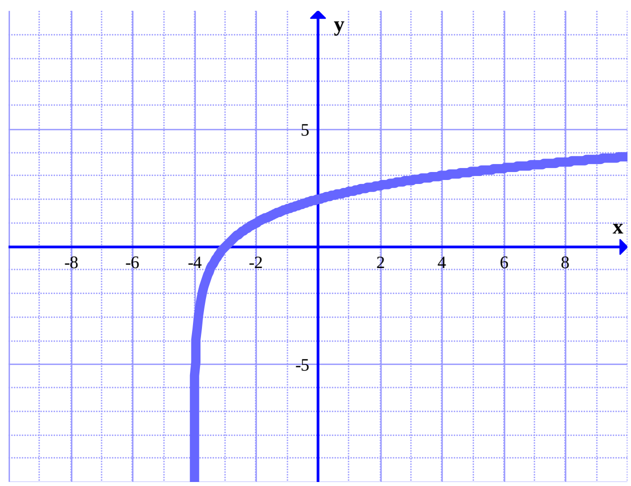


Domain $(0, \infty)$

x	y	computations
1	0	(y=0) $2^0 = x$ 1 = x
½	-1	(y = -1) $2^{-1} = x$ ½ = x
2	1	(y=1) $2^1 = x$ 2 = x
¼	-2	(y=-2) $2^{-2} = x$ ¼ = x
4	2	(y=2) $2^2 = x$ 4 = x

53) $y = \log_2(x+4)$

$2^y = x + 4$ (problem written in exponential form)
Domain $(-4, \infty)$



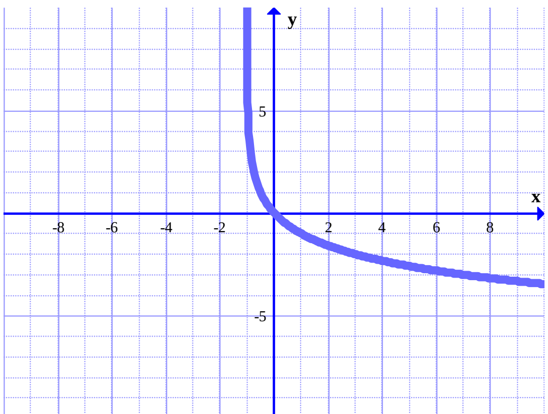
x	y	computations
-3	0	(y = 0) $2^0 = x + 4$ 1 = x + 4 -3 = x
-3.5	-1	(y=-1) $2^{-1} = x$ ½ = x + 4 .5 = x+4 -3.5 = x
-2	1	(y =1) $2^1 = x+4$ 2 = x + 4 -2 = x
-3.75	-2	(y = -2) $2^{-2} = x+4$ ¼ = x+4 0.25 = x+4 -3.75 = x
0	2	(y=2) $2^2 = x+4$ 4 = x+4 0 = x

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.4

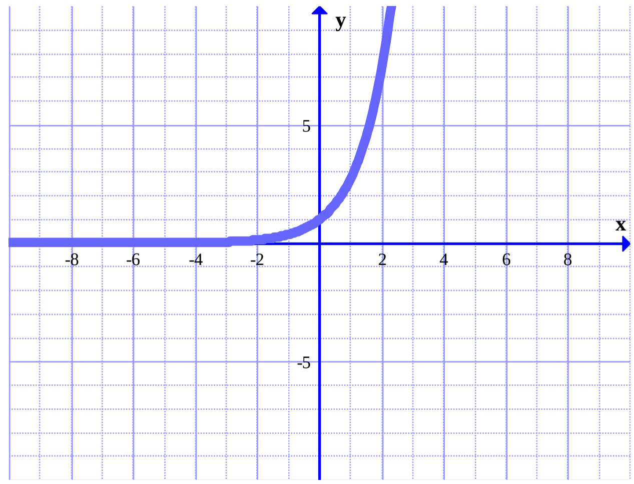
57) $y = log_{1/2}(x + 1)$

equation written in exponential form
$(1/2)^y = x + 1$
Domain $(-1, \infty)$



x	y	computations
0	0	(y = 0) $(1/2)^0$ = x+1 1 = x+1 0 = x
1	-1	(y = -1) $(1/2)^{-1}$ = x+ 1 $(2/1)^1$ = x+ 1 2 = x+1 1 = x
-1/2	1	(y=1) $(1/2)^1$ = x+1 ½ = x+1 ½-1 = x $\frac{1}{2} - \frac{2}{2} = x$
3	-2	(y = -2) $(1/2)^{-2}$ = x+1 $(2/1)^2$ = x+1 4 =x+1 3=x
-3/4	2	(y=2) $(1/2)^2$ = x+1 ¼ = x+1 ¼ - 1 = x $\frac{1}{4} - \frac{4}{4} = x$ -3/4 = x

13) y= e^x (from 6.5)

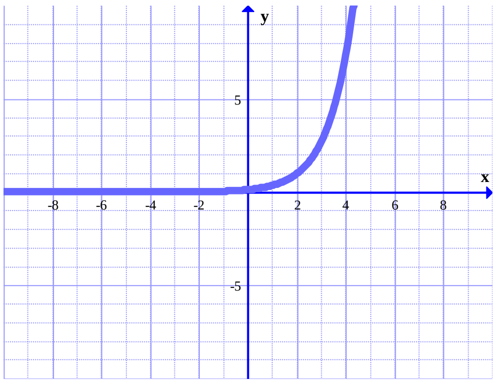


x	Y	computations – done on calculator
0	1	y=e^0 y = 1
-1	.37	y=e^{-1} =.37
1	2.72	y = e^1 y = 2.72
-2	.14	y = e^{-2} y = .14
2	7.39	y = e^2 y=7.39

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.5

17) $y = e^{x-2}$



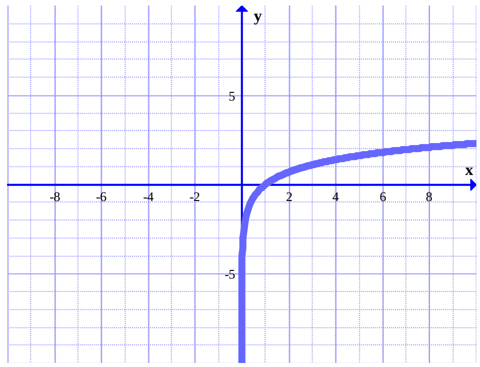
x	Y	computations – done on calculator
2	0	$y = e^{2-2}$ $y= e^{0}$ $y = 1$
3	2.72	$y = e^{3-2}$ $y = e^{1}$ $y = 2.72$
1	.37	$y= e^{1-2}$ $y = e^{-1}$ $y = 0.37$
4	7.39	$y= e^{4-2}$ $y = e^{2}$ $y = 7.39$
0	.14	$y=e^{0-2}$ $y = e^{-2}$ $y = 0.14$

19) $y = \ln(x)$

Equation written in exponential form:

$e^{y} = x$

Domain $(0, \infty)$

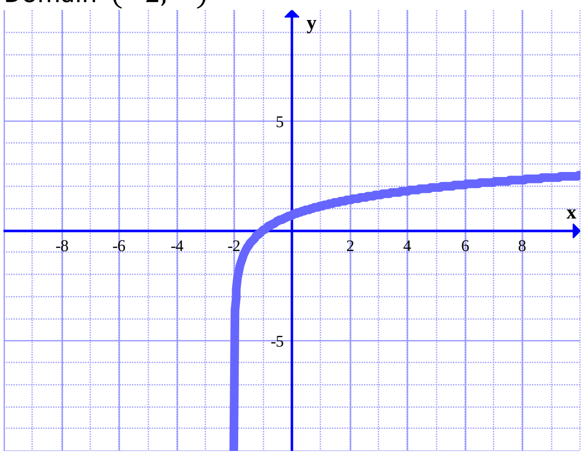


x	y	computations- done on calculator
1	0	$(y=0)\ e^{0} = x$ $1 = x$
2.72	1	$(y = 1)\ e^{1} = x$ $2.72 = x$
.37	-1	$(y = -1)\ e^{-1} = x$ $0.37 = x$
7.39	2	$(y = 2)\ e^{2} = x$ $7.39 = x$
.14	-2	$(y = -2)\ e^{-2} = x$ $0.14 = x$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

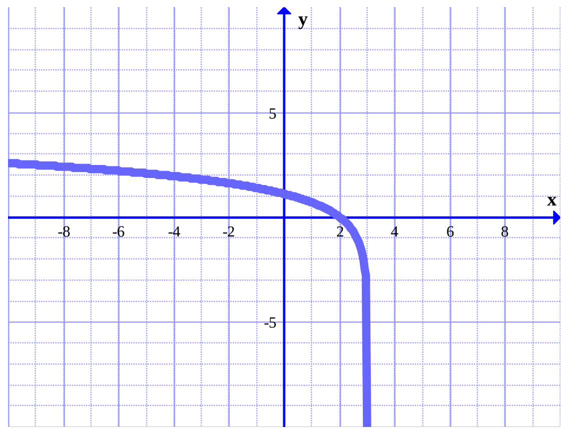
Section 6.5

23) $y = \ln(x+2)$
Equation written in exponential form:
$e^y = x+2$
Domain $(-2, \infty)$



x	y	computations, done on calculator
-1	0	(y=0) $e^0 = x+2$ 1 = x+2 -1 = x
.72	1	(y=1) $e^1 = x+2$ 2.72 = x+2 x = .72
-1.63	-1	(y=-1) $e^{-1} = x+ 2$. .37 = x+ 2 -1.63 = x
5.39	2	(y = 2) $e^2 = x+2$ 7.39 = x+2 5.39 =x
-1.86	-2	(y=-2) $e^{-2} = x+2$ 0.14 = x+2 -1.86 = x

25) $y = \ln(3 - x)$
Equation written in exponential form:
$e^y = 3 - x$
I will rewrite a bit more
$x = 3 - e^y$
Domain $(-\infty, 3)$



x	y	computations- done on calculator
2	0	(y=0) $x = 3-e^0 = 2$
2.63	-1	(y=-1) $x = 3-e^{-1} = 2.63$
.28	1	(y=1) $x = 3-e^1 = 3-2.72 = .28$
2.86	-2	(y=-2) $x = 3-e^{-2} = 3-.14 = 2.86$
-4.39	2	(y=2) $x = 3-e^2 = 3 - 7.39 = -4.39$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.5

29) An initial investment of $10,000 earns 5.25% interest compounded continuously. What will the investment be worth in 8 years?
P = 10000
r = .0525
t = 8
Substitute values in formula
Use a calculator to evaluate.
$A=10000e^{8*.0525} = 15219.62$
Solution: $15219.62

Section 6.6

1) $\log_2 16$

This is asking the question $2^? = 16$, the answer is 4, because $2^4 = 16$

Solution: 4

5) $\log_8 8^5$

This is asking the question $8^? = 8^5$, the answer is 5, because $8^5 = 8^5$

Solution: 5

9) ln(e)
Think of this as $\log_e e$
This is asking the question $e^? = e$, the answer is 1, because $e^1 = e$

Solution: 1

11) $\log_2 64^3$
This is asking the question $2^? = 64^3$
I will need to do a bit of algebra to solve this. I know $2^6 = 64$. I can rewrite the problem.
$2^? = (2^6)^3$ I
will clear the parenthesis by multiplying exponents
$2^? = 2^{18}$
Now it should be obvious that 18 is the solution.
Solution: 18

Section 6.6

13) Which of these is a true statement?

a) If I replace with the values to the left:

b) If I replace with the values above:

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

a) $\log_2(4*8) = \log_2 4 * \log_2 8$
b) $\log_2(4*8) = \log_2 4 + \log_2 8$

To answer these questions I need to know what each part equals.

$\log_2(4*8) = \log_2 32 = 5$

$\log_2 4 = 2$

$\log_2 8 = 3$

$\log_2(4*8) = \log_2 4 * \log_2 8$

5 = 2*3
5 = 6

This is false.

$\log_2(4*8) = \log_2 4 + \log_2 8$
5 = 2 + 3
5 = 5 (this is true

Solution: a is false, b is true

15) Which of these is a true statement?

a) $log_2 \frac{16}{2} = \frac{log_2 16}{log_2 2}$

b) $log_2 \frac{16}{2} = log_2 16 - log_2 2$

I need to know what each piece is equal to, before I can answer the question.

$log_2 \frac{16}{2} = log_2 8 = 3$

$\log_2 16 = 4$

$\log_2 2 = 1$

a) If I replace with the values above:

$log_2 \frac{16}{2} = \frac{log_2 16}{log_2 2}$

$3 = \frac{4}{1}$ (this is false)

b) If I replace with the values above:

$log_2 \frac{16}{2} = log_2 16 - log_2 2$

3 = 4 - 1

3=3 (this is true)

Solution: a is false, b is true

Section 6.6

17) Which of these is a true statement?
a) $\log 100^3 = (\log 100)^3$
b) $\log 100^3$ = = $3\log 100$

a) I will replace the values to determine if this is true or not.

$\log 100^3 = (\log 100)^3$

b) I will replace the values to determine if this is true or not.

$\log 100^3$ = = $3\log 100$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

I need to compute each piece before I can answer the question. You can use your calculator if you can't do these by hand yet.

$\log 100^3 = 6$

$\log 100 = 2$

$6 = (2)^3$
$6 = 8$ (this is false)

$6 = 3*2$
$6 = 6$ (this is true)

Solution: a is false, b is true

21) $\log_5(25x^2y^6)$

$= \log_5 25 + \log_5 x^2 + \log_5 y^6$
$= 2 + 2\log_5 x + 6\log_5 y$

Solution: $\log_5 25 + 2\log_5 x + 6\log_5 y$
or $2 + 2\log_5 x + 6\log_5 y$

23) $\log_2 \frac{xy^3}{z^2}$

$= \log_2(xy^3) - \log_2 z^2$

$= \log_2 x + \log_2 y^3 - \log_2 z^2$

$= \log_2 x + 3\log_2 y - 2\log_2 z$

Solution: $\log_2 x + 3\log_2 y - 2\log_2 z$

25) $\log_2 \frac{xy}{w^2z^5}$

$= \log_2(xy) - \log_2(w^2z^5)$

$= \log_2 x + \log_2 y - (\log_2 w^2 + \log_2 z^5)$

$= \log_2 x + \log_2 y - 2\log_2 w - 5\log_2 z$

Solution: $\log_2 x + \log_2 y - 2\log_2 w - 5\log_2 z$

29) $\log_2(x^2 \cdot \sqrt[3]{y})$

$= \log_2 x^2 + \log_2 \sqrt[3]{y}$

$= 2\log_2 x + \log_2 y^{1/3}$

$= 2\log_2 x + \frac{1}{3}\log_2 y$

Solution: $2\log_2 x + \frac{1}{3}\log_2 y$

Section 6.6

33) $2\log_3 x + 4\log_3 y + \log_3 z$

$= \log_3 x^2 + \log_3 y^4 + \log_3 z$

35) $5\log_2 x + 3\log_2 y - \log_2 z$

$= \log_2 x^5 + \log_2 y^3 - \log_2 z$

$= \log_3 x^3y^4z$

Solution: $\boldsymbol{\log_3 x^3y^4z}$

$=\log_2(x^5y^3) - \log_2 z$

$= \log_2 \frac{x^5y^3}{z}$

Solution: $\boldsymbol{\log_2 \frac{x^5y^3}{z}}$

37) $4\log x - 2\log y - 3\log z$

$=\log x^4 - (2\log y + 3\log z)$

$= \log x^4 - (\log y^2 + \log z^3)$

$=\log x^4 - \log(y^2z^3)$

$= \log \frac{x^4}{y^2z^3}$

Solution: $\boldsymbol{\log \frac{x^4}{y^2z^3}}$

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

3) $2^{x+1} = 32$
$2^{x+1} = 2^5$

$x+1 = 5$

$x = 4$
This solution will check, but I am not showing the work.
Solution: x = 4

5) $\left(\frac{1}{2}\right)^x = 16$

$\frac{1}{2} = 2^{-1}$ I need to use this fact to solve this problem without logarithms.

$(2^{-1})^x = 2^4$

$2^{-x} = 2^4$

$-x = 4$

$x = -4$
Solution: x = -4

Check x = -4 $\left(\frac{1}{2}\right)^{-4} = 16$ $\left(\frac{2}{1}\right)^4 = 16$ $16 = 16$ Th e solution checks. x = -4 is the only solution

7) $2^{4-x} = 64$
$2^{4-x} = 2^6$
$4 – x = 6$
$-x = 6 – 4$
$-x = 2$
$x = -2$

Solution: x = -2

Check x = -2 $2^{4-(-2)} = 64$ $2^{4+2} = 64$ $2^6 = 64$ $64 = 64$ The solution checks. x = -2 is the only solution.

9) $32^x = 2$
$(2^5)^x = 2^1$
$2^{5x} = 2^1$
$5x = 1$
$x = 1/5$

Solution: x = 1/5

Check x = 1/5 $32^{1/5} = 2$ $\sqrt[5]{32} = 2$ $2 = 2$ The solution checks. x = 1/5 is the only solution.

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

13) $3^x = 6$
take log of both sides: $\log 3^x = \log 6$
make exponent coefficient: $x\log 3 = \log 6$
divide: $x = \frac{\log 6}{\log 3}$ or $\log_3 6$
(I will get a decimal approximation using my calculator)

Solution: $x = \log_3 6$
approx: 1.63

Check x = 1.63 (because I am checking a rounded answer, this might not check quite as nicely as the last few problems.

$3^{1.63} = 6$
5.9938 = 6 (this isn't exactly 6=6, but it is close enough)

15) $e^x = 12$
I will take the ln of both sides, because of the e in the problems.

take ln of both sides: $\ln(e^x) = \ln(12)$
make exponent coefficient: $x\ln e = \ln(12)$
substitute ln(e) =1: $x(1) = \ln(12)$
$x = \ln(12)$
x= 2.48

Solution: $x = \ln(12)$
approx: 2.48

Check x = 2.48

$e^{2.48} = 12$
11.941 = 12 (this is still close enough because I am using a rounded answer)

19) $32e^{2x} = 128$
first divide by 32 : $\frac{32e^{2x}}{32} = \frac{128}{32}$

$e^{2x} = 4$
I will take the ln of both sides: $\ln(e^{2x}) = \ln 4$

make exponent coefficient: $2x\ln e = \ln 4$

replace lne with 1: $2x = \ln 4$

divide: $x = \frac{\ln 4}{2}$

(This solution will check, I am not going to show how this solution checks)

Solution: $x = \frac{\ln(4)}{2}$
approx: .69

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

25) $\log_3 x = 2$
rewrite in exponential form:
$3^2 = x$
$9 = x$
Solution: x = 9

Check x = 9 $\log_3 9 = 2$ (you can evaluate the logarithm on your calculator, if you can't do it in your head) 2 = 2 The solution checks and x = 9 is the only solution.

27) ln x = 1

rewrite with $\ln = \log_e$

$\log_e x = 1$

write without log

$e^1 = x$

Solution: x = e (approx 2.72)

check x = e ln e = 1 1 = 1 The solution checks. x = e is the only solution.

31) $\log_4(3x-6) = -1$
write in exponential form
$4^{-1} = 3x-6$
$\frac{1}{4} = \frac{3x-6}{1}$ (cross multiply)
4(3x-6) = 1*1
12x – 24 = 1
12x = 25
x = 25/12
Solution: x = 25/12

check x = 4 $\log_4 64 = 3$ (again, use your calculator if you can't do the log in your head) 3 = 3 The solution checks. x = 4 is the only solution.

35) $\log_2(x-1) = 3$
write without log
$2^3 = x - 1$
8 = x – 1
9 = x

Solution: x = 9

Check x = 9 $\log_2(9-1) = 3$ $\log_2 8 = 3$ (again, use your calculator if you can't do the log in your head) 3 = 3 The solution checks. x = 3 is the only solution.

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

37) $\log_2(2x)=5$
write without the logarithm

$2^5 = 2x$

32 = 2x

16 = x

(This solution checks, but I will not include the checking here.)

Solution: x = 16

39) log(x+1) = log(3x-2)

you can solve be dropping the logs and equating the arguments.

x+1 = 3x -2
-2x = -3
$= x = \frac{-3}{-2} = \frac{3}{2}$

Solution: x = 3/2

Check x = 3/2

$$log\left(\frac{3}{2}+1\right) = log\left(3*\frac{3}{2}-2\right)$$

$$log\left(\frac{3}{2}+\frac{2}{2}\right) = log\left(\frac{9}{2}-\frac{4}{2}\right)$$

$$log\left(\frac{5}{2}\right) = log\left(\frac{5}{2}\right)$$

The solution checks.
x=3/2 is the only solution.

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

43) $\log_2 x - \log_2(x+6) = -2$

write as a single log $\quad log_2 \frac{x}{x+6} = -2$

write without the log, in exponential form
$2^{-2} = \frac{x}{x+6}$
replace 2^{-2} with 1/4: $\quad \frac{1}{4} = \frac{x}{x+6}$

cross multiply $\quad 4x = x + 6$
$3x = 6$
$x = 2$

Solution: x = 2

Check x = 2

$\log_2 2 - \log_2(2+6) = -2$
$1 - \log_2 8 = -2$
$1-3 = -2$
$-2 = -2$

The solution checks.
x = -2 is the only solution.

45) $\log_2 x - \log_2 (x - 6) = 2$

write a single log: $log_2 \frac{x}{x-6} = 2$

write in exponential form: $2^2 = \frac{x}{x-6}$

$4 = \frac{x}{x-6}$

clear fractions: $4(x-6) = \frac{x}{x-6}(x-6)$

$4x - 24 = x$
$3x = 24$
$x = 8$

Solution: x = 8

Check x = 8

$\log_2 8 - \log_2(8-6) = 2$

$3 - \log_2 2 = 2$

$3 - 1 = 2$

$2 = 2$

The solution checks.
x= 8 is the only solution

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

47) $\log_2(x+6) - \log_2(3x+2) = -1$

write as a single log: $log_2 \frac{x+6}{3x+2} = -1$

write in exponential form: $2^{-1} = \frac{x+6}{3x+2}$

replace 2^{-1} with 1/2": $\frac{1}{2} = \frac{x+6}{3x+2}$

cross multiply: 1(3x+2) = 2(x+6)

Solve: 3x + 2 = 2x + 12

x = 10

This solution will check. I am not including the work.
Solution: x = 10

51) $\log_3(x+6) + \log_3(3x) = 4$

write as a single log: $log_3(x+6)(3x) = 4$

simplify: $\log_3(3x^2 + 18x) = 4$

write in exponential form: $3^4 = 3x^2 + 18x$

$81 = 3x^2 + 18x$

divide by 3: $\frac{81}{3} = \frac{3x^2}{3} + \frac{18x}{3}$

$27 = x^2 + 6x$
$0 = x^2 + 6x - 27$
0 = (x+9)(x-3)
x+9 = 0 or x – 3 =0
x = -9, or x =3

Solution: x =3
x = -9 is extraneous

Check x = 3 $\log_3(3+6) + \log_3(3*3) = 4$ $\log_3 9 + \log_3 9 = 4$ 2 + 2 = 4 The solution checks. x = 3 is the only solution.	Check x = -9 $\log_3(-9+6)+\log_3(3*(-9)) = 4$ $\log_3(-3) + \log_3(-27) = 4$ Each of these logs is a non-real number. The solution does not check x = -9 is extraneous

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

Use the formula: $P = P_0e^{kt}$, where P_0 is the initial population at t=0, and k is the rate of growth.

57) The bacterial in a laboratory culture increased from an initial population of 500 to 1,500 in 3 hours. How long will it take the population to reach 10,000? (hint first use the 500 to 1,500 in 3 hours and the formula above to solve for k, then use the 10,000 to answer the question.) (round t the nearest hour)

Use the first group of numbers and the formula to solve for k.

P = 1500 P_0 = 500   t = 3

Substitute values into equation: $1500 = 500e^{k*3}$

(divide by 500) $3 = e^{3k}$

(take the ln of both sides) $\ln(3) = \ln(e^{3k})$

(make the exponent a coefficient) $\ln(3) = 3k\ln(e)$

(substitute lne = 1) $\ln(3) = 3k$

divide by 3 $\frac{\ln(3)}{3} = k$

Now use the next bit of information to answer the question.

P = 10000

P_0 = 500

k = $\frac{\ln(3)}{3}$

continued in right column

57) continued:

put the values in the formula $10000 = 500e^{ln3/3t}$

(divide by 500) $20 = e^{ln3/3t}$

(take the ln of both sides) $\ln(20) = lne^{ln3/3t}$

(make exponent a coefficient) $\ln(20) = \frac{ln3}{3}t(lne)$

(replace lne with 1) $\ln(20) = \frac{\ln(3)}{3}t$

(multiply by $\frac{3}{\ln(3)}$) $\frac{3}{\ln(3)}\ln(20) = \frac{3}{\ln(3)} * \frac{\ln(3)}{3}t$

(use your calculator) $\frac{3}{\ln(3)}\ln(20) = t$

8.1805 = t

Solution: About 8 hours

Chapter 6: Exponential and Logarithmic Functions
Selected Solutions to Odd Problems

Section 6.7

Use the continuous compound interest formula: $A = Pe^{rt}$, where P is the initial investment, r the annual interest rate, and t the number of years to solve the following.

61) How long will it take an initial investment of \$1,000 to double if it is expected to earn 6% interest compounded continuously? (round to 1 decimal place)

find values for the formula
P = 1,000
A = 2,000 (goal is to double the initial investment.
r = .06

Substitute these values into the formula.
$2000=1000e^{.06t}$

(divide by 1000) $2 = e^{.06t}$

(take ln of both sides) $\ln(2) = \ln(e^{.06t})$

(make exponent coefficient)
$\ln(2) = (.06t)\ln e$

($\ln e = 1$) $\ln(2) = .06t$

divide: $\frac{\ln(2)}{.06} = t$

(use calculator)
11.6 = t

Solution: about 11.6 years

63) How long will it take an initial investment of \$10,000 to grow to \$15,000 if it is expected to earn 4% interest compounded continuously? (round to 1 decimal place)

find values for the formula:

A = 15000
P = 10000
r = .04

substitute into formula
$15000= 10000e^{.04t}$

(divide by 10000) $1.5 = e^{.04t}$

(take ln of each side) $\ln(1.5) = \ln(e^{.04t})$

(make exponent a coefficient)
$\ln(1.5) = .04t\ln(e)$

($\ln(e) = 1$) $\ln(1.5) = .04t$

(divide) $\frac{\ln(1.5)}{.04} = t$

(use calculator)
t = 10.1
Solution: about 10.1 years