Grima MAT 151

Chapter 6 Practice Test

1) $f(x) = x^2 - 2x + 1$ $g(x) = 7x - 5$

Find the following:

a) $(g \circ f)(x)$ $= 7(x^2-2x+1)-5$

$= 7x^2-14x+7-5$

$= \boxed{7x^2-14x+2}$

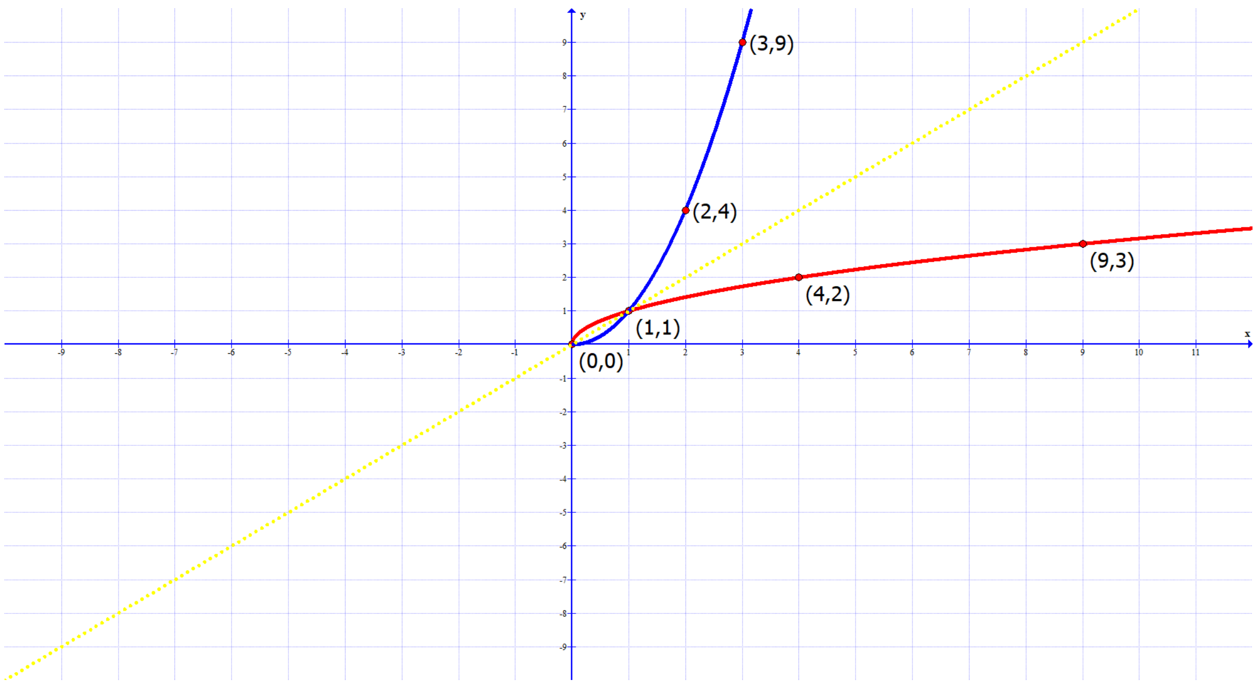
b) the domain of $(g \circ f)(x)$

$\boxed{(-\infty, \infty)}$

- Hint for part a: This is the same as g(f(x)) and it requires replacing any x in the right side of the g-function with the entire right side of the f-function.
- Hint for part b: Since both g and f are polynomials there will be no algebra required to find this domain. In fact if you do any algebra you will likely find an x-intercept and it will not belong as part of a correct answer.

2) The graph of a one to one f function is given. Draw the graph of the inverse function f^{-1}.

(Just switch the x and y values of each point, plot the new points and connect them with the same shape.)



3) f(x) = x^3 + 4

a) Find the inverse of f(x) $\boxed{f^{-1}(x) = \sqrt[3]{x-4}}$

b) Check your answer by showing that $(f \circ f^{-1})(x)$ = x

Steps:

1) Change the function notation to “y” $y = x^3 + 4$
2) Switch “x” and “y” $x = y^3 + 4$
3) Solve for “y” $-4 \quad -4$ $\quad \sqrt[3]{x-4} = \sqrt[3]{y^3}$ $\quad \sqrt[3]{x-4} = y$
4) Replace the “y” with a inverse function symbol $f^{-1}(x)$ ($usually\ f^{-1}(x),$) $f^{-1}(x) = \sqrt[3]{x-4}$
5) Move on to part “b” and check you answer by finding $(f \circ f^{-1})(x)$ (answer is correct only when this reduces to “x”)

How to find: $(f \circ f^{-1})(x)$

- Write the “f” function without the x and leave a parenthesis where the x was. $(f \circ f^{-1})(x) = (\sqrt[3]{x-4})^3 + 4$
- Insert the right side of the "f^{-1}" function inside the parenthesis.
- Simplify, my answer to part a is wrong if this simplifies to anything other than just a single “x”.

$= x - 4 + 4$

$= x$

4) $f(x) = e^x$

i) Find the requested function.

ii) Describe the transformation compared to f(x)

Hint:

- $f(x + h)$ shifts left h
- $f(x – h)$ shifts right h
- $f(x) + h$ shifts up h
- $f(x) – h$ shifts down h
- $-f(x)$ reflects over x-axis
- $f(-x)$ reflects over y-axis)

a) $f(x – 2)$

i) $f(x-2) = \log_2(x-2)$

ii) Right 2

b) $f(x) + 4$

i) $f(x)+4 = \log_2(x) + 4$

ii) up 4

c) $-f(x)$

i) $-f(x) = -\log_2(x)$

ii) Reflect over x-axis

d) $f(x + 3) - 2$

i) $f(x+3)-2 = \log_2(x+3) - 2$

ii) left 3 down 2

#5-7: Solve

5) $3^{x+2} = 81$

- Prime factor the 81 to write both sides with the same base
- Drop the bases, set the exponents equal
- Solve for x
- Check

$$3^{x+2} = 3^4$$

$$\begin{array}{r} x+2 = 4 \\ -2 \;\; -2 \\ \hline x = 2 \end{array}$$

Check

$$3^{2+2} = 81$$

$$81 = 81$$

6) $\left(\frac{1}{2}\right)^{x+1} = \frac{1}{16}$

- Prime factor the 16
- Write each side with a negative exponent
- Multiply exponents to clear the parenthesis on the left side
- Drop the bases and set the exponents equal
- Solve for x
- Check

$$\left(\frac{1}{2^1}\right)^{x+1} = \frac{1}{2^4}$$

$$\left(2^{-1}\right)^{x+1} = 2^{-4}$$

$$2^{-1x-1} = 2^{-4}$$

$$-1x-1 = -4$$
$$\underline{\quad +1 \quad +1}$$
$$\frac{-1x}{-1} = \frac{-3}{-1}$$
$$x = 3$$

Check

$$\left(\frac{1}{2}\right)^{3+1} = \frac{1}{16}$$

$$\frac{1}{16} = \frac{1}{16}$$

7) $4^{3x+1} * 4^{2x-3} = 4^{18}$

- Simplify the left side by adding the exponents (the 4 will remain the base)
- Drop the bases and set the exponents equal
- Solve for x
- Check

$$4^{3x+1+2x-3} = 4^{18}$$

$$4^{5x-2} = 4^{18}$$

$$\begin{aligned} 5x-2 &= 18 \\ +2 \quad &\; +2 \\ \hline 5x &= 20 \\ \hline \end{aligned}$$

$$\boxed{x = 4}$$

Check

$$4^{3(4)+1} * 4^{2(4)-3} = 4^{18}$$

$$4^{13} * 4^{5} = 4^{18}$$

$$4^{13+5} = 4^{18}$$

$$4^{18} = 4^{18}$$

8 - 9: Let $f(x) = \log_2(x)$

8) $f(x + 1) \quad + 3$

a) Find the requested function. $\log_2(x+1) \quad +3$

b) State the domain of function created in part a. $(-1, \infty)$

c) Describe the transformation compared with f(x) left 1

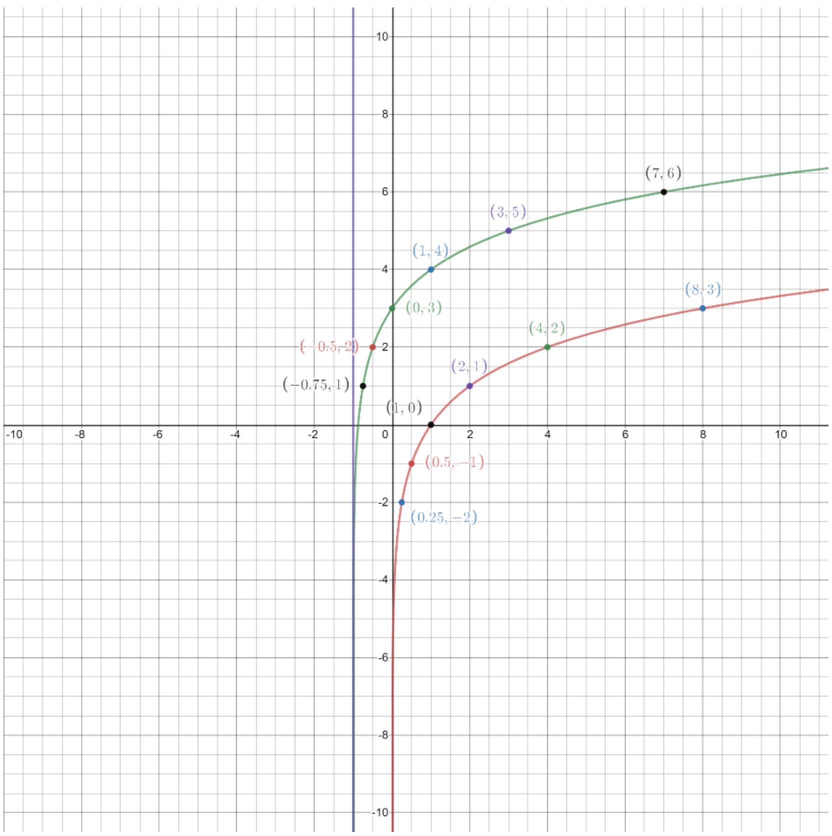
up 3

d) Graph the logarithmic function

$x+1 < 0$

$x < -1$

Here is a graph of $f(x) = \log_2(x)$ to help you out



Here are the points that are marked

x	f(x)
.25	-2
.5	-1
1	0
2	1
4	2

8 - 9: Let $f(x) = \log_2(x)$

9) $f(x - 3) - 2$

a) Find the requested function. $\log_2(x-3) - 2$

b) State the domain of function created in part a. $(3, \infty)$

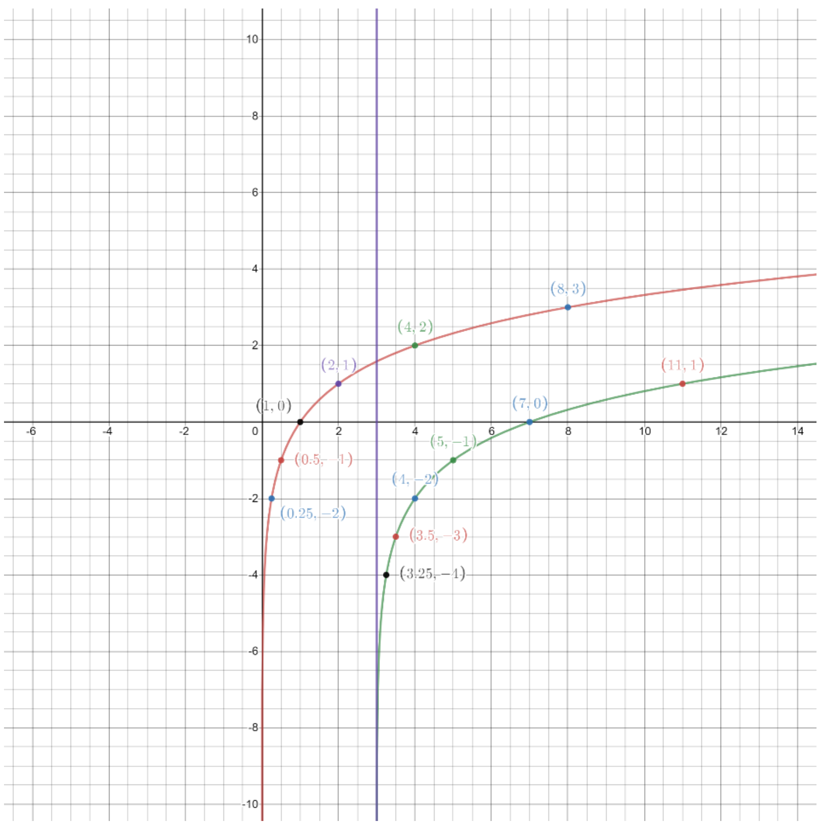
c) Describe the transformation compared with f(x)

d) Graph the logarithmic function

↑ Right 3
Down 2

$x - 3 > 0$
$+3 \quad +3$
$x > 3$

Here is a graph of $f(x) = \log_2(x)$ to help you out



Here are the points that are marked

x	f(x)
.25	-2
.5	-1
1	0
2	1
4	2

10) Write the expression as a single logarithm. Write your answer with only positive exponents.

$2\log_3 x + 4\log_3 y - 5\log_3 z$

Steps:

1) Make any coefficient an exponent of the logarithm it is in front of.

$= \log_3 x^2 + \log_3 y^4 - \log_3 z^5$

2) Write a single log with the base that is common.

Fraction with only z^5 in the Denominator

3) Place the arguments in the correct place, with the correct signs between them.

- There should not be any plus or minus signs used on this step.
- If there is no subtraction, then there is no fraction.
- If there is a minus sign, any argument whose logarithm is preceded by a minus sign belongs in the denominator of a fraction. The rest of the arguments belong in the numerator.

$$= \log_3 \frac{x^2 y^4}{z^5}$$

11) Expand into sums and differences of logarithms (express exponents as multiplication). $log_3 \frac{x^2 y}{w^4 z}$

Steps:

1) Write one logarithm with the appropriate base for each letter (variable). In this case we will have 4 logarithms as there are 4 variables.

$= \log_3 x^2 \quad \log_3 y \quad \log_3 w^4 \quad \log_3 z$

2) Put the appropriate sign between the logarithms. You may only use plus or minus signs on this step. If you use a times or divide sign you have made a mistake.

Minus in front

- Put a minus sign in front of any logarithm whose variable is in the denominator, the rest get plus signs in front, or no sign in front if it is the first logarithm in your answer.

Since in denominator

3) Make each exponent a coefficient.

$= \log_3 x^2 + \log_3 y - \log_3 w^4 - \log_3 z$

$= 2\log_3 x + \log_3 y - \log_3 w^4 - \log_3 z$

#12-13: Solve the exponential equations, round your answer to 2 decimals.

12) $3^x = 18$ $\qquad \log_3 18 = x$

Steps:

[redacted]

$$x = \log_3 18 \approx 2.63$$

13) [redacted] $3^{x-5} = 2$

Steps:

[redacted]

$$\log_3 2 = x - 5$$

$$+5 \qquad\qquad +5$$

$$5 + \log_3 2 = x$$

$$(5.63)$$

#14-19: Solve the logarithmic equations, round to 2 decimals when needed.

14) $\log_3 x = 4$

$3^4 = x$

Steps:

1) Rewrite the problem in exponential form.

$\boxed{81 = x}$

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

Check

$\log_3 81 = 4$

$4 = 4$ ✓

15) ln x = 2

$\log_e x = 2$

$\boxed{e^2 = x}$

Steps:

1) Change $\ln(x)$ to $log_e(x)$
2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

3) Solve for x
4) Check

Check

$\ln e^2 = 2$

$2 = 2$ ✓

16) $\log_2(x+1) = 5$

$2^5 = x+1$

$32 = x+1$

$-1 \quad -1$

$\boxed{31 = x}$

Steps:

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

Check

$\log_2(31+1) = 5$

$5 = 5$ ✓

17) ln (4x-8) = ln(3x-1)

Steps:

1) Drop the logs and set the arguments equal to each other.

2) Solve for x.

3) Check.

$$\begin{array}{rcl} 4x-8 &=& 3x-1 \\ -3x+8 & & -3x+8 \\ \hline \end{array}$$

$$\boxed{x=7}$$

Check

$$\ln(4(7)-8) = \ln(3(7)-1)$$

$$\ln(20) = \ln(20)$$

18) $\log_2(x+2) - \log_2(x-2) = 1$

$$\log_2 \frac{x+2}{x-2} = 1$$

$$2^1 = \frac{x+2}{x-2}$$

Steps:

1) Use the minus to divide rule to write the left side with one logarithm.

$$\frac{2}{1} = \frac{x+2}{x-2}$$

2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base to of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.)

3) Solve for x (you will need to cross multiply to solve)

4) Check

$$2(x-2) = 1(x+2)$$

$$2x-4 = 1x+2$$

$$-1x+4 \quad -1x+4$$

$$x = 6$$

Check

$$\log_2(6+2) - \log_2(6-2) = 1$$

$$1 = 1 \checkmark$$

19) $\log_2 (x+2)+\log_2 (x-2) = 5$
(be sure to check for extraneous solutions)

Steps:

1) Use the plus to times rule to write the left side with one logarithm.
2) Simplify the argument by performing the multiplication.
3) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

4) Solve for x (you will need set to equal zero and solve by factoring)
5) Check

$\log_2 (x+2)(x-2) = 5$

$\log_2 (x^2 - 2x + 2x - 4) = 5$

$\log_2 (x^2 - 4) = 5$

$2^5 = x^2 - 4$

$32 = x^2 - 4$

$-32 \quad\quad -32$

$0 = x^2 - 36$

$0 = (x+6)(x-6)$

$x = -6$ DOES NOT check

$x = 6$ DOES check

$\boxed{x=6}$

Check $x=-6$

$\log_2(-6+2) + \log_2(-6-2) = 5$

Domain Error $= 5$

Check $x = 6$

$\log_2(6+2) + \log_2(6-2) = 5$

$5 = 5$ ✓