Grima MAT 151
Chapter 8 Practice Test

#1-2: Solve each system of equations using either the substitution method or the elimination method, 0 points if no work is shown even if answer is correct.

1) $\frac{2}{3}x + \frac{1}{4}y = 3$

$x = y - 1$

Answer $(3,4)$

$\frac{2}{3}(y-1) + \frac{1}{4}y = 3$

$\frac{4}{4}\cdot\frac{2}{3}y - \frac{2}{3} + \frac{3}{3}\cdot\frac{1}{4}y = \frac{3}{1}\cdot\frac{3}{3}$

$\frac{8}{12}y + \frac{3}{12}y - \frac{2}{3} = \frac{9}{3}$

$\frac{11}{12}y \quad 2/3 = 9/3$

Fractions are not your friends for this type of problem. Rewrite any equation that has a fraction to an equivalent equation without a fraction before starting the steps below.

Steps to solve a system of equations using the **substitution method:**

1) Choose an equation to use for the substitution.

2) If needed, solve the equation picked in step 1 for a variable (try not to create any fractions).

3) Substitute the equation created in step 2 into the unused equation.

   - If at any time all the variables drop out stop doing Algebra and follow the steps at the bottom of the page.

4) Solve

5) Substitute the answer created in step 4 into the equation created in step 2.

6) Write your answer.

7) Check

$\frac{11}{12}y - \frac{2}{3} = \frac{9}{3}$

$+\frac{2}{3} \quad +\frac{2}{3}$

$\frac{12}{11}\cdot\frac{11}{12}y = \frac{\cancel{11}^{\,1}}{\cancel{3}_{\,1}}\cdot\frac{\cancel{12}^{\,4}}{\cancel{11}_{\,1}}$

$y = 4$

$x = y - 1$

$x = 4 - 1$

$x = 3$

$\boxed{(3,4)}$

2) $\begin{pmatrix} 3x+2y=13 \\ x-5y=-7 \end{pmatrix}$ 5, 2

$15x+10y=65$
$2x-10y=-14$

$17x = 51$

$\frac{17x}{17} = \frac{51}{17}$

$x=3$

Answer (3,2)

Steps to solve a system of equations using the **Elimination method:**

1) Line up the variables. (This step will be done for us for most of the problems.)

2) Multiply one or possibly both equation(s) by a suitable number(s) so that the two equations have the same variable with opposite coefficients.

3) Add the equations together to drop out a variable.

4) Solve the equation created in step 3.

5) Substitute the answer created in step 4 into any equation from the problem.

6) Write your answer.

7) Check

$3x+2y=13$
$3(3)+2y=13$
$9+2y=13$
$-9 \quad -9$

$2y=4$

$\frac{2y}{2}=\frac{4}{2}$

$y=2$

$\boxed{(3,2)}$

3) Solve each system of equations, by hand without matrices, 0 points if no work is shown even if answer is correct.

$$2x + 4y - 5z = 17$$
$$-x + y + 2z = -5$$
$$x - 3y + 3z = -2$$

(pair the middle equation with the other 2 and drop out the x's)

Answer: $(4, 1, -1)$

General solution strategy:

1) Pick an equation pair it with the other two. This creates two pairs of equations.

2) Pick a letter to drop (don't pick the z) then drop the same letter from each pair. This will create two equations with two unknowns.

3) Take the two equations created in the last step and solve them using the elimination method. This will give answers for 2 of the 3 variables.

4) Substitute the answers from part 3 into one of the original equations and solve for the remaining variable. Write your solution (x,y,z) but use numbers for the x, y and z.

$2(-x + y + 2z = -5)$

$2x + 4y - 5z = 17$

$-2x + 2y + 4z = -10$

$2x + 4y - 5z = 17$

$6y - 1z = 7$

$-x + y + 2z = -5$

$x - 3y + 3z = -2$

$-2y + 5z = -7$

$6y - 1z = 7$

$3(-2y + 5z = -7)$ →

$6y - 1z = 7$

$-6y + 15z = -21$

$\frac{14z}{14} = \frac{-14}{14}$

$z = -1$

$6y - 1z = 7$

$6y - 1(-1) = 7$

$6y + 1 = 7$

$6y = 6$

$y = 1$

$x - 3y + 3z = -2$ → $x - 3(1) + 3(-1) = -2$

$x - 3 - 3 = -2$

$x - 6 = -2$

$+6 \quad +6$

$x = 4$

$\boxed{(4, 1, -1)}$

4) Solve the system of equations using matrices and row operations. 0 points if no matrix work is shown even if answer is correct.

$$3x + 2y = 16$$
$$2x - 3y = -11$$

$$\left[\begin{array}{cc|c} 3 & 2 & 16 \\ 2 & -3 & -11 \end{array}\right]$$

Answer: (2,5)

Step 1) Create the matrix implied by the system of equations.

$$\begin{array}{l} 2R1 \\ -3R2 \end{array} \quad \begin{array}{cc|c} 6 & 4 & 32 \\ -6 & 9 & 33 \\ \hline 0 & 13 & 65 \\ \hline 13 & 13 & 13 \end{array}$$

$$\begin{array}{cc|c} 0 & 1 & 5 \end{array}$$

Step 2: Use elimination method to eliminate the x's.

Step 3: divide away the common factor

Step 4: Make the answer to step 2 the bottom row of the matrix

Step 5: Use the elimination method to eliminate the y's.

Step 6: divide away the common factor:

2nd matrix $\left[\begin{array}{cc|c} 3 & 2 & 16 \\ 0 & 1 & 5 \end{array}\right]$

Step 7: Make the answer to step 6 the new top row for the matrix created in step 4

Step 8: Create the system of equations from the matrix created in step 6.

Step 9: Simplify the equations and write your answer.

$$\begin{array}{l} R1 \\ -2R2 \end{array} \quad \begin{array}{cc|c} 3 & 2 & 16 \\ 0 & -2 & -10 \\ \hline \frac{3}{3} & \frac{0}{3} & \frac{6}{3} \end{array}$$

$$\begin{array}{cc|c} 1 & 0 & 2 \end{array}$$

Step 10: Check

$$\boxed{(2,5)}$$

Final matrix $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \end{array}\right]$

#5-6 Use the following matrices to answer all the problems in this section.

A = $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 6 & 1 \end{bmatrix}$	B = $\begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$	C = $\begin{bmatrix} 1 & 0 & -1 \\ 7 & 2 & 4 \end{bmatrix}$
D = $\begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix}$		

5) 2D – C

$$2\begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -1 \\ 7 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 & 0 \\ 8 & -2 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -1 \\ 7 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1 & 4-0 & 0-(-1) \\ 8-7 & -2-2 & 6-4 \end{bmatrix}$$

Answer: $\begin{bmatrix} 5 & 4 & 1 \\ 1 & -4 & 2 \end{bmatrix}$

A = $\begin{bmatrix}1 & 0\\3 & 2\\6 & 1\end{bmatrix}$	B = $\begin{bmatrix}4 & 5\\1 & 2\end{bmatrix}$	C = $\begin{bmatrix}1 & 0 & -1\\7 & 2 & 4\end{bmatrix}$
D = $\begin{bmatrix}3 & 2 & 0\\4 & -1 & 3\end{bmatrix}$		

6) AC

$$\begin{bmatrix}1 & 0\\3 & 2\\6 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\7 & 2 & 4\end{bmatrix}$$

$$\begin{matrix}1\\0\end{matrix}\cdot\begin{matrix}1\\7\end{matrix} = \begin{array}{c}1\\0\\\hline 0\end{array} \qquad \begin{matrix}1\\0\end{matrix}\cdot\begin{matrix}0\\2\end{matrix} = \begin{array}{c}0\\0\\\hline 0\end{array} \qquad \begin{matrix}1\\0\end{matrix}\cdot\begin{matrix}-1\\4\end{matrix} = \begin{array}{c}-1\\0\\\hline -1\end{array}$$

Top Row

$$\begin{matrix}3\\2\end{matrix}\cdot\begin{matrix}1\\7\end{matrix} = \begin{array}{c}3\\14\\\hline 17\end{array} \qquad \begin{matrix}3\\2\end{matrix}\cdot\begin{matrix}0\\2\end{matrix} = \begin{array}{c}0\\4\\\hline 4\end{array} \qquad \begin{matrix}3\\2\end{matrix}\cdot\begin{matrix}-1\\4\end{matrix} = \begin{array}{c}-3\\8\\\hline 5\end{array}$$

Answer: $\begin{bmatrix}1 & 0 & -1\\17 & 4 & 5\\13 & 2 & -2\end{bmatrix}$

Middle Row

7) has been deleted

$$\begin{matrix}6\\1\end{matrix}\cdot\begin{matrix}1\\7\end{matrix} = \begin{array}{c}6\\7\\\hline 13\end{array} \qquad \begin{matrix}6\\1\end{matrix}\cdot\begin{matrix}0\\2\end{matrix} = \begin{array}{c}0\\2\\\hline 2\end{array} \qquad \begin{matrix}6\\1\end{matrix}\cdot\begin{matrix}-1\\4\end{matrix} = \begin{array}{c}-6\\4\\\hline -2\end{array}$$

Bottom Row

8) Solve the system of equations using Cramer's rule, 0 points if solved with another method, even if answer is correct.

$$3x - 2y = 4$$
$$2x + 3y = 7$$

$D = \begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix}$

$= 3 \cdot 3 \quad - 2(-2)$

$= 9 + 4$

$D = 13$

D = 13

D_x = 26

D_y = 13

X = 2 y = 1

$X = \frac{D_x}{D} = \frac{26}{13} = 2$

$y = \frac{D_y}{D} = \frac{13}{13} = 1$

Here is how to use Cramer's rule (minus the misspelling of his name)

Crammer's Rule 2x2

$x - y = 4$ $\quad x = ?$ $\quad x = \frac{Dx}{D}$ $\quad y = \frac{Dy}{D}$

$2x + y = 2$ $\quad y = ?$

$D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3$ $\quad x = \frac{Dx}{D} = \frac{6}{3} = 2$

$Dx = \begin{vmatrix} 4 & -1 \\ 2 & 1 \end{vmatrix} = 6$ $\quad y = \frac{Dy}{D} = \frac{-6}{3} = -2$

$Dy = \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} = -6$

$Dx = \begin{vmatrix} 4 & -2 \\ 7 & 3 \end{vmatrix}$

$= 4 \cdot 3 - 7(-2)$

$= 12 + 14$

$D_x = 26$

$Dy = \begin{vmatrix} 3 & 4 \\ 2 & 7 \end{vmatrix}$

$= 3 \cdot 7 - 2 \cdot 4$

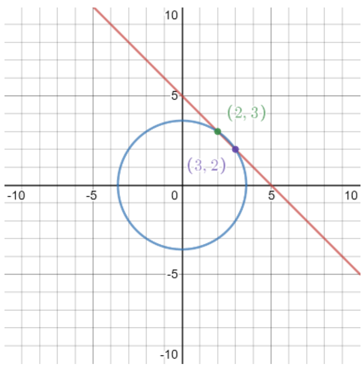
$= 21 - 8$

$Dy = 13$

#9-10: Solve the following systems of equations.

9) $x + y = 5$
$x^2 + y^2 = 13$

Answer (2,3) (3,2)



$$X + y = 5$$
$$-y \quad -y$$
$$X = 5 - y$$

$$(5-y)^2 + y^2 = 13$$
$$(5-y)(5-y) + y^2 = 13$$
$$25 - 5y - 5y + y^2 + y^2 = 13$$
$$2y^2 - 10y + 25 = 13$$
$$-13 \quad -13$$
$$2y^2 - 10y + 12 = 0$$
$$2(y^2 - 5y + 6) = 0$$
$$2(y-2)(y-3) = 0$$

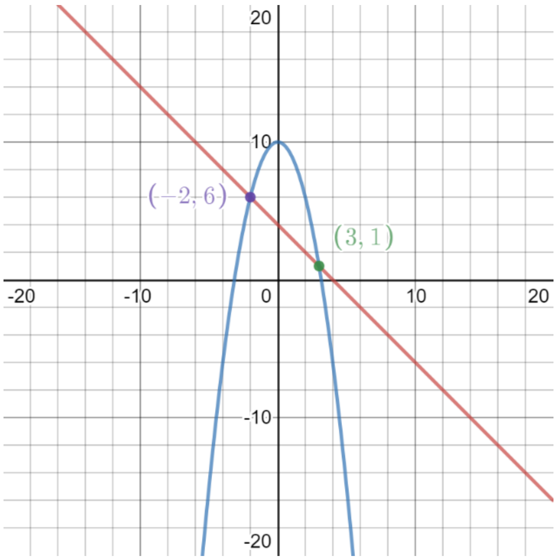
$2 = 0$
NO SOLUTION

$$y - 2 = 0 \qquad y - 3 = 0$$
$$y = 2 \qquad y = 3$$

$$X = 5 - y \qquad X = 5 - y$$
$$X = 5 - 2 \qquad X = 5 - 3$$
$$X = 3 \qquad X = 2$$
$$(3, 2) \qquad (2, 3)$$

10) $x + y = 4$
$x^2 + y = 10$

Answer $(-2,6)$ $(3,1)$



$$x + y = 4$$
$$-x \qquad -x$$
$$y = 4 - x$$

$$x^2 + y = 10$$
$$x^2 + 4 - x = 10$$
$$-10 \qquad -10$$
$$x^2 - x - 6 = 0$$
$$(x+2)(x-3) = 0$$

$x + 2 = 0$ $\qquad$ $x - 3 = 0$

$x = -2$ $\qquad$ $x = 3$

$y = 4 - x$ $\qquad$ $y = 4 - 3$

$y = 4 - (-2)$ $\qquad$ $y = 1$

$y = 6$ $\qquad$ $(3,1)$

$(-2,6)$

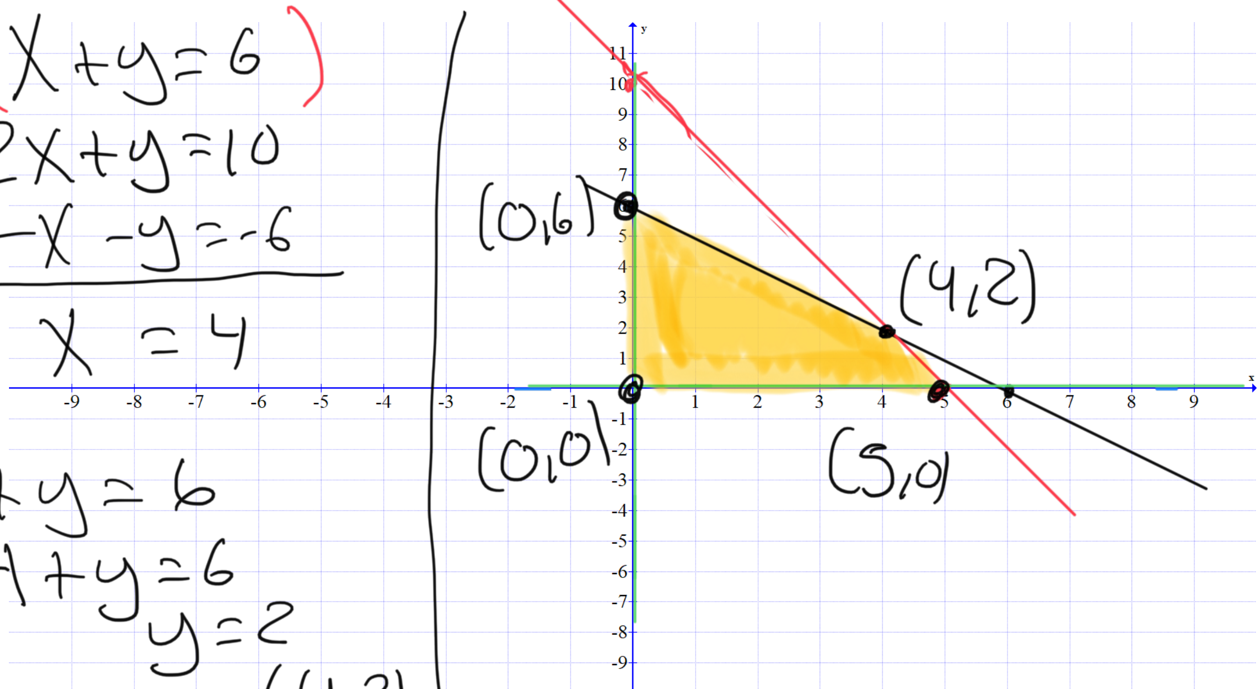
11) graph the system of linear inequalities by hand. Label the corner points.

$x + y \leq 6$
$2x + y \leq 10$
$x \geq 0, \quad y \geq 0$

Answer: *corner points* (4,2) (0,0) (0,6) (5,0)

-1 (X+y=6)
2X+y=10
-X -y=-6
X = 4

X+y=6
4+y=6
y=2
(4,2)



X+y=6
0+y=6
y=6
(0,6)

X+0=6
X=6
(6,0)
Shade Down

2X+y=10
2(0)+y=10
y=10
(0,10)

2X+0=10
2X=10
X=5
(5,0)
Shade Down

X≥0
yAXIS
Shade
Right

y≥0
XAXIS
Shade
UP