Chapter 5 Practice Test (Complete all problems)

#1-4: Find the following antiderivatives, be sure to include the plus "C" in your answer.

1) $\int 8x^3 dx$

$$= 8\int x^3 dx$$

1st: $\int af(x)dx = a\int f(x)dx$

$$= \overset{2}{\cancel{8}} \cdot \frac{1}{\underset{1}{\cancel{4}}} x^4 + C$$

2nd: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

$$= 2x^4 + C$$

Answer: $2x^4 + C$

2) $\int (8x^3 - 6x^2 + 5)dx$

1st:

$$= \int 8x^3 dx - \int 6x^2 dx + \int 5 dx$$

$\int (f(x) + g(x))dx = \int f(x)dx \quad + \int g(x)dx$

$$= 8\int x^3 dx - 6\int x^2 dx + \int 5 dx$$

$\int (f(x) - g(x))dx = \int f(x)dx \quad - \int g(x)dx$

$$= \cancel{8}^{2} \cdot \frac{1}{\cancel{4}} x^4 - \cancel{6}^{2} \cdot \frac{1}{\cancel{3}} x^3 + 5x + C$$

2nd: $\int af(x)dx = a\int f(x)dx$

$$= 2x^4 - 2x^3 + 5x + C$$

3rd:

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int a dx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

$answer$: $2x^4 - 2x^3 + 5x + C$

3) $\int \frac{4}{x^3} dx$

1st: Rewrite with negative exponent

$$= \int 4x^{-3} dx$$

2nd: $\int af(x)dx = a \int f(x)dx$

$$= 4 \int x^{-3} dx$$

$$= \overset{-2}{\cancel{4}} \cdot \frac{1}{\cancel{-2}_{1}} x^{-2} + C$$

3rd: Power Rule: $\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$

$$= -2x^{-2} + C$$

4th: rewrite with positive exponent

$$= \frac{-2}{x^2} + C$$

$answe: -\frac{2}{x^2} + C$

4) $\int \frac{7}{x} dx$

1st: Rewrite with -1 exponent $= \int 7x^{-1} dx$

$= 7 \int x^{-1} dx$

2nd: $\int af(x)dx = a \int f(x)dx$

$= 7 \ln|x| + C$

3rd: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

answer $7ln|x| + C$

#5 – 10: Use u-substitution to evaluate the indefinite integrals.

5) $\int 2x(x^2+5)^2 dx$

$= \int (x^2+5)^2\, 2x\,dx$

Rewrite the problem so that the parenthesis is first:

let $u = x^2+5x$

Next: $let\ u = inside\ of\ the\ parenthesis$

$\int u^2\, 2x\,dx$

Rewrite the problem so that the “parenthesis is changed to an "u"

$u = x^2+5$

Next find $\frac{du}{dx}$

$dx\,\frac{du}{dx} = 2x\,dx$

Multiply by dx to clear the fraction.

$du = 2x\,dx$

$\int u^2\,du$

Next replace to make problem only have u’s

$= \frac{1}{3}u^3 + C$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Last change $u\ back\ to\ get\ the\ answer$

$= \frac{1}{3}(x^2+5)^3 + C$

$answer$: $\frac{1}{3}(x^2+5)^3 + C$

6) $\int 15x^2 e^{5x^3}\,dx$

Rewrite the problem so that the "e" is written first: $= \int e^{5x^3}\, 15x^2\, dx$

Next: $let\ u = exponent\ of\ the\ e$

let $u = 5x^3$

Rewrite the problem so that the exponent is changed to an "u" $= \int e^{u}\, 15x^2\, dx$

$u = 5x^3$

Next find $\frac{du}{dx}$

$dx\,\frac{du}{dx} = 15x^2\,dx$

Multiply by dx to clear the fraction.

$du = 15x^2\,dx$

Next replace to make problem only have u's $= \int e^{u}\,du$

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$ $= e^{u} + C$

Last change $u\ back\ to\ get\ the\ answer$ $= e^{5x^3} + C$

$answer$: $e^{5x^3} + C$

7) $\int 6x(x^2+5)^2 dx$ $= \int (x^2+5)^2 6x\,dx$

Rewrite the problem so that the parenthesis with the exponent is first:

$u = x^2+5$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^2 6x\,dx$

Rewrite the problem so that the “parenthesis with the exponent is changed to an "u"

$u = x^2+5$

Next find $\frac{du}{dx}$ $dx\frac{du}{dx} = 2x\,dx$

Multiply by dx to clear the fraction. $3du = 3\cdot 2x\,dx$

$3du = 6x\,dx$

This is not good enough. Multiply to make a perfect match

$= \int u^2 3du$

Next replace to make problem only have u’s

$= 3\int u^2 du$

Next integrate: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ $provided\ \overset{n}{\cancel{x}} \neq -1$

$= \cancel{3}\cdot\frac{1}{\cancel{3}}u^3 + C$

$answer\ \ (x^2+5)^3 + C$

$= 1u^3 + C$

$= u^3 + C$

$= (x^2+5)^3 + C$

8) $\int 6xe^{x^2}dx$

Rewrite the problem so that the "e" is written first: $= \int e^{x^2} 6x\,dx$

Next: $let\ u = exponent\ of\ the\ e$ $\quad$ let $u = x^2$

$= \int e^{u} 6x\,dx$

Rewrite the problem so that the exponent is changed to an "u"

$u = x^2$

Next find $\frac{du}{dx}$

$dx\frac{du}{dx} = 2x\,dx$

Multiply by dx to clear the fraction.

$3\,du = 3 \cdot 2x\,dx$ $\qquad = \int e^{u} 3\,du$

This is not good enough. Multiply to make a perfect match.

$3\,du = 6x\,dx$

Next replace to make problem only have u's $\qquad = 3\int e^{u}\,du$

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$

$= 3e^{u} + C$

Last change $u\ back\ to\ get\ the\ answer$

$= 3e^{x^2} + C$

$answer\ \ 3e^{x^2} + C$

9) $\int \frac{4}{4x+1} dx$ $= \int 4(4x+1)^{-1} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

$= \int (4x+1)^{-1} \cdot 4 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = 4x+1$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^{-1} \cdot 4 dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = 4x+1$

Next find $\frac{du}{dx}$

$dx \frac{du}{dx} = 4 dx$

Multiply by dx to clear the fraction.

$du = 4 dx$ $= \int u^{-1} du$

Next replace to make problem only have u's

$= \ln|u| + C$

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

$= \ln|4x+1| + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer\ \ ln|4x+1| + C$

10) $\int \frac{9}{3x+7} dx$ $= \int 9(3x+7)^{-1} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

$= \int (3x+7)^{-1} 9 \cdot dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = 3x+7$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^{-1} 9 dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = 3x+7$

Next find $\frac{du}{dx}$

$dx \frac{du}{dx} = 3dx$

Multiply by dx to clear the fraction.

$3\ du = 3 \cdot 3dx$

This is not good enough. Multiply to make a perfect match.

$3du = 9dx$

$\int u^{-1} 3du$

Next replace to make problem only have u's

$= 3 \int u^{-1} du$

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

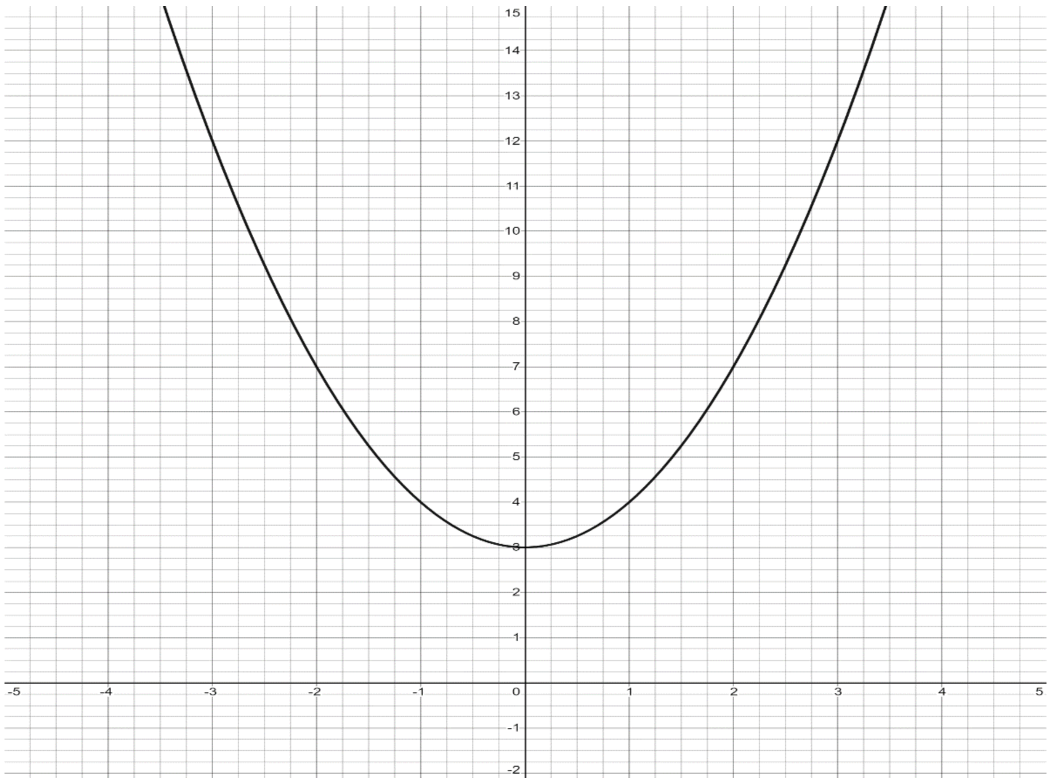
$= 3\ln|3x+7| + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer\ \ 3ln|3x+7| + C$

11) Follow the instructions and create rectangles on the provided graph, or one that you create (using right endpoints) to estimate the area between the curve and the x-axis.

$f(x) = x^2 + 3$; from a = 0 to b = 3 using 3 rectangles



11a) Determine the width of each rectangle that will be used to estimate the area.

(Each rectangle will have a width; $\frac{b-a}{n}$)

$width = 1$

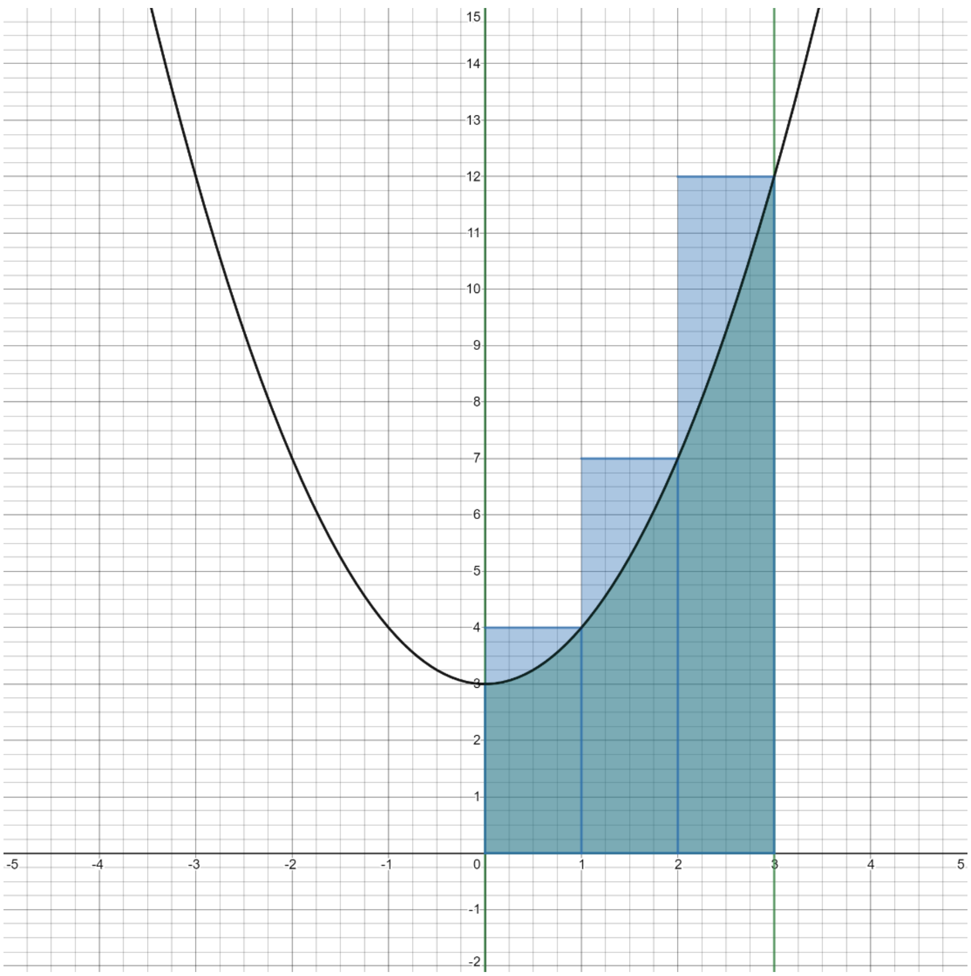
$\frac{3-0}{3} = \frac{3}{3} = 1$ (w.o.m)

11b) Increment by the interval width and draw rectangles on the graph that is provided. (next page)

Rectangle	Starting and ending x-values
1	(0, 1)
2	(1,2)
3	(2,3)

11c) Find the area of each rectangle. (Let the height of the rectangle be the function value at the right endpoint of the rectangle.) (areas are 4, 7, and 12)

Rectangle	Starting and ending x-values	Length (height) $f(x) = x^2$	Width	Area
1	(0, 1)	$f(1) = 1^2 + 3 = 4$	$1 - 0 = 1$	$A = 4 * 1 = 4$
2	(1,2)	$f(2) = 2^2 + 3 = 7$	$2 - 1 = 1$	$A = 7 * 1 = 7$
3	(2,3)	$f(3) = 3^2 + 3 = 12$	$3 - 2 = 1$	$A = 12 * 1 = 12$



11d) Add the individual areas to get an estimate of the area under the curve (23)

$4 + 7 + 12 = 23$

#12-15: Use the Fundamental Theorem of Calculus to evaluate the definite integral.

12) $\int_1^4 (2x-4)dx$

$= \int_1^4 2x\,dx - \int_1^4 4\,dx$

1st:

$\int (f(x) - g(x))dx = \int f(x)dx \; - \int g(x)dx$

$= 2\int_1^4 x - \int_1^4 4\,dx$

2nd: $\int af(x)dx = a\int f(x)dx$

$= 2 \cdot \frac{1}{2}x^2 - 4x \Big|_1^4$

3rd:

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \;\; provided \; n \neq -1$

$= x^2 - 4x \Big|_1^4$

Integral of a constant Rule: $\int a dx \; = \; ax + C \;\; (a \; is \; any \; real \; number)$

4th Evaluate the integral at ~~2~~ 4 then at 1

$4^2 - 4(4) = 0$

$1^2 - 4(1) = -3$

$= 0 - (-3)$

$= 3$

5th subtract the results to get the answer

$answer \;\; 3$

13) $\int_0^3 (2x)(x^2+1)^2 dx$

$\int (x^2+1)^2\, 2x\, dx$

Rewrite the problem so that the parenthesis is first:

let $u = x^2+1$

$\int u^2\, 2x\, dx$

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis is changed to an "u"

$u = x^2+1$

Next find $\frac{du}{dx}$

$dx\frac{du}{dx} = 2x\,dx$

$= \int u^2\, du$

Multiply by dx to clear the fraction.

$du = 2x\,dx$

$= \frac{1}{3}u^3$

Not good enough, multiply to make a perfect match

Next replace to make problem only have u's

$= \frac{1}{3}(x^2+1)^3 \Big|_0^3$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

$\frac{1}{3}(3^2+1)^3 = \frac{1}{3}\cdot 1000 = \frac{1000}{3}$

$\frac{1000}{3} - \frac{1}{3}$

Last change $u\ back\ to\ compute\ the\ integral$

Evaluate the integral at 3 then at 0

$\frac{1}{3}((0)^2+1)^2 = \frac{1}{3}$

$= \frac{999}{3}$

subtract the results to get the answer

$= 333$

$answer$: 333

14) $\int_1^e \frac{2}{x}dx$ $= \int_1^e 2x^{-1}dx$

1st: Rewrite with -1 exponent

$= 2\int_1^e x^{-1}dx$

2nd: $\int af(x)dx = a\int f(x)dx$

$= 2\ln|x| \Big|_1^e$

3rd: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

4th Evaluate the integral at e then at 1

$2\ln|e| = 2(1) = 2$

5th subtract the results to get the answer

$2\ln|1| = 2(0) = 0$

$= 2 - 0$

$= 2$

answer: 2

15) $\int_0^3 4(2x-1)^2\,dx$ $= \int (2x-1)^2 \cdot 4\,dx$

Rewrite the problem so that the parenthesis is first: let $u = 2x-1$

Next: $let\ u = inside\ of\ the\ parenthesis$ $= \int u^2 \cdot 4\,dx$

Rewrite the problem so that the "parenthesis is changed to an "u"

$u = 2x-1$

Next find $\frac{du}{dx}$ $dx\frac{du}{dx} = 2dx$

Multiply by dx to clear the fraction. $2du = 2 \cdot 2dx$

$2du = 4dx$

Not good enough, multiply to make a perfect match $= \int u^2\, 2du$

Next replace to make problem only have u's $= 2\int u^2 du$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ $provided\ n \neq -1$ $= 2 \cdot \frac{1}{3} u^3$

Last change $u\ back\ to\ compute\ the\ integral$ $= \frac{2}{3}(2x-1)^3 \Big|_0^3$

Evaluate the integral at ~~2~~ 3 then at ~~1~~ 0

$\frac{2}{3}(2(3)-1)^3 = \frac{250}{3}$

subtract the results to get the answer

$\frac{2}{3}(2(0)-1)^3 = -\frac{2}{3}$

$= \frac{250}{3} - -\frac{2}{3}$

$answer\ \ 84$

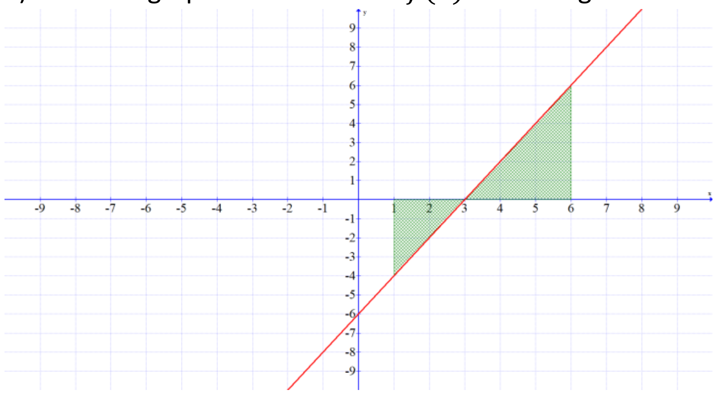
$= \frac{250}{3} + \frac{2}{3}$

$= 252/3$

$= 84$

16) $f(x) = 2x - 6$; $[1,6]$

a) Sketch a graph of the function $f(x)$ over the given interval $[a, b]$.



$$2x - 6 = 0$$
$$2x = 6$$
$$x = 3$$

b) Find any x-intercept within the interval $[a, b]$.

$(3, 0)$

c) Find the area between the x-axis and f(x) over the interval $[a, b]$ using definite integrals. (Find this area by hand)

$\left|\int_1^3 (2x - 6)dx\right|$ (region beneath x-axis)

$$\left| x^2 - 6x \Big|_1^3 \right| = \left| -9 - (-5) \right| = \left| -9 + 5 \right|$$
$$= \left| -4 \right|$$
$$= 4$$

$$3^2 - 6(3) = -9$$
$$1^2 - 6(1) = -5$$

$\int_3^6 (2x - 6)dx$ (region above the x-axis)

$$= 2\int_3^6 x\,dx - \int_3^6 6\,dx = 2 \cdot \frac{1}{2} x^2 - 6x \Big|_3^6$$
$$= x^2 - 6x \Big|_3^6$$

$$(6)^2 - 6(6) = 0$$
$$(3)^2 - 6(3) = -9$$

$$= 0 - (-9)$$
$$= 2$$

$\boxed{answer: 4 + 9 = 13}$

17)

The function whose graph is represented by the dashed is $f(x) = 2x + 5$

The function whose graph is represented by the solid graph is $g(x) = x^2 + 2$

a) Create the integral needed to find the shaded area $\int_{-1}^{2}(-x^2 + 2x + 3)\,dx$

$\int_{-1}^{3}(2x+5)-(x^2+2)\,dx = \int_{-1}^{3}(2x+5-x^2-2)$

b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area) $\boxed{answer\ 10.67}$