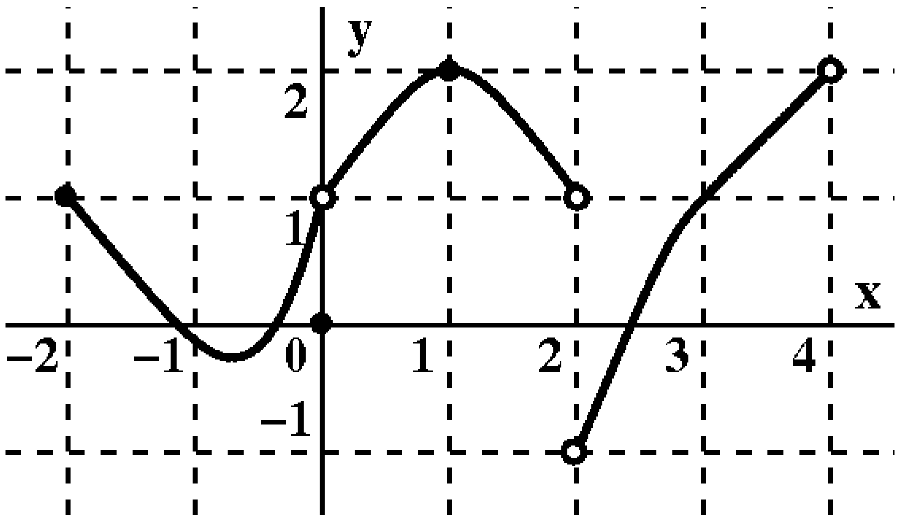
Chapter 1 Practice test Part 1 (should complete all of the problems)

1) Below is a graph of the function f(x).



Find the following.

1a) f(0) $= 0$ y-coordinate of (0,0)

1b) f(1) $= 2$ y-coordinate of (1,2)

1c) f(-2) $= 1$ y-coordinate of (-2,1)

1d) $\lim_{x\to 0^-} f(x) = 1$ y-coordinate of (0,1)

1e) $\lim_{x\to 0^+} f(x) = 1$ y-coordinate of (0,1)

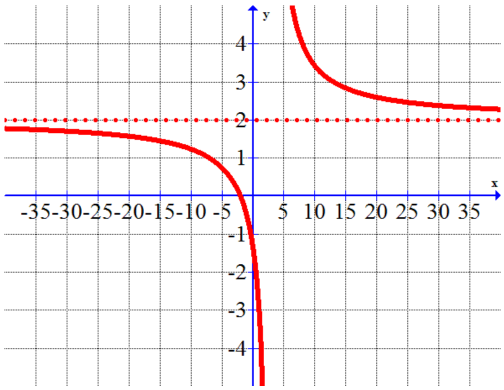
1f) $\lim_{x\to 0} f(x) = 1$ y-coordinate of (0,1)

1g) $\lim_{x\to 2^-} f(x) = 1$ y-coordinate of (2,1)

1h) $\lim_{x\to 2^+} f(x) = -1$ y-coordinate of (2,-1)

1i) $\lim_{x\to 2} f(x) = dne$ dne since 1g and 1h are not equal

2)

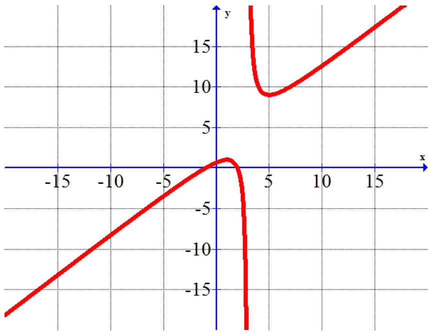


Graph flattens out at the horizontal asymptote of $y = 2$ in both directions. Thus limits are equal to 2.

2a) $\lim_{x\to\infty} f(x) = 2$

2b) $\lim_{x\to-\infty} f(x) = 2$

3)



a) $\lim_{x \to \infty} f(x) = \infty$ (graph approaches top of y-axis)

b) $\lim_{x \to -\infty} f(x) = -\infty$ (graph approaches bottom of y-axis)

4) Find the following limits using Algebra.

4a) $\lim_{x \to 2}(x^2 + 4x - 3)$ $= (2)^2 + 4(2) - 3 = 4 + 8 - 3 = 9$

4a) $\lim_{x \to 2}(x^2 + 4x - 3) = 9$

4b) $\lim_{x \to -2} \frac{x^2+5x+6}{x^2+8x+12}$

$= \lim_{x \to -2} \frac{(x+2)(x+3)}{(x+2)(x+6)}$

= $\lim_{x \to -2} \frac{x+3}{x+6} = \frac{-2+3}{-2+6} = \frac{1}{4}$

4b) $\lim_{x \to -2} \frac{x^2+5x+6}{x^2+8x+12} = \frac{1}{4}$

4c) $\lim_{x \to 49} \frac{\sqrt{x}-7}{x-49}$

$$\lim_{x \to 49} \frac{\sqrt{x}-7}{x-49}$$

$$= \lim_{x \to 49} \frac{\sqrt{x}-7}{x-49} * \frac{\sqrt{x}+7}{\sqrt{x}+7}$$

$$= \lim_{x \to 49} \frac{x+7\sqrt{x}-7\sqrt{x}-49}{(x-49)(\sqrt{x}+7)}$$

$$= \lim_{x \to 49} \frac{x-49}{(x-49)(\sqrt{x}+7)}$$

$$= \lim_{x \to 49} \frac{1}{\sqrt{x}+7}$$

$$= \frac{1}{\sqrt{49}+7}$$

$$= \frac{1}{7+7}$$

$$\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} = 1/14$$

4c) $\lim_{x \to 49} \frac{\sqrt{x}-7}{x-49} = \frac{1}{14}$

5) Find the following limits using Algebra.

a) $\lim_{x\to\infty} \frac{8x^2+1}{2x^2+4x}$

$= \lim_{x\to\infty} \frac{1/x^2}{1/x^2}\left(\frac{8x^2+1}{2x^2+4x}\right)$

$= \lim_{x\to\infty} \frac{\frac{1}{x^2}*8x^2+\frac{1}{x^2}*1}{\frac{1}{x^2}*2x^2+\frac{1}{x^2}*4x}$

$= \lim_{x\to\infty} \frac{8+\frac{1}{x^2}}{2+\frac{4}{x}}$

$= \frac{8+\frac{1}{\infty^2}}{2+\frac{4}{\infty}}$

$= \frac{8+0}{2+0}$

$\lim_{x\to\infty} \frac{8x^2+1}{2x^2+4x} = 4$

b) $\lim_{x\to\infty} \frac{5x-4}{2x^2-x+2}$

$= \lim_{x\to\infty} \frac{1/_{x^2}}{1/x^2}\left(\frac{5x-4}{2x^2-x+2}\right)$

$= \lim_{x\to\infty} \frac{\frac{1}{x^2}*5x-\frac{1}{x^2}*4}{\frac{1}{x^2}*2x^2-\frac{1}{x^2}*x+\frac{1}{x^2}*2}$

$= \lim_{x\to\infty} \frac{\frac{5}{x}-4/x^2}{2-\frac{1}{x}+\frac{2}{x^2}}$

$= \frac{\frac{5}{\infty}-\frac{4}{\infty^2}}{2-\frac{1}{\infty}+\frac{2}{\infty^2}}$

$= \frac{0+0}{2-0+0}$

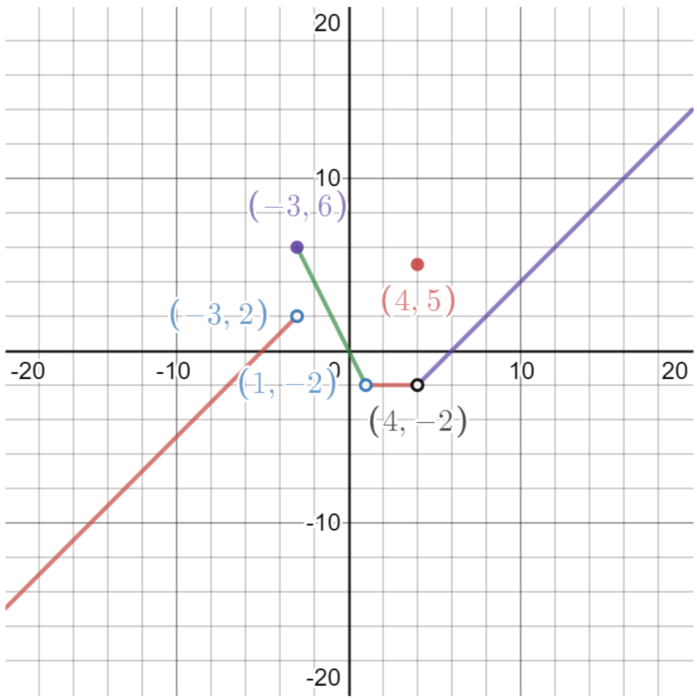
$= \frac{0}{2} = 0$

$\lim_{x\to\infty} \frac{5x-4}{2x^2-x+2} = 0$

5a) $\lim_{x\to\infty} \frac{8x^2+1}{2x^2+4x} = 4$

5b) $\lim_{x\to\infty} \frac{5x-4}{2x^2-x+2} = 0$

6) Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous.



$x = -3$
$x = 1$
$x = 4$

7) Find all values of $x = a$ where the function $f(x)$ is discontinuous. State if the function is continuous everywhere. You do not need to state the reason the function is discontinuous. If the function is continuous everywhere simply answer the function is continuous everywhere.

7a) $f(x) = \frac{x+3}{x^2+4x-5}$

$x^2 + 4x - 5 = 0$
$(x + 5)(x - 1) = 0$
$x = -5, x = 1$ (both values have asymptotes as neither factor cancels with the numerator.)

Not continuous at $x = -5,1$

7b) $f(x) = 5x + 10$

Function is continuous everywhere (no Algebra required for polynomials)

8) Find the average rate of change for each function over the given interval. It is not necessary to sketch a graph to model the average rate of change.

$f(x) = x^3 + 5x$ between $x = 1$ and $x = 2$

$f(1) = 1^3 + 5(1) = 6$
$point\ (1,6)$

$f(2) = 2^3 + 5(2) = 18$
$point\ (2, 18)$

$average\ rate\ of\ change = \frac{18-6}{2-1} = \frac{12}{1} = 12$

Average rate of change = 12

9) f(x) = $x^2 + 3$

a) Use the definition of the derivative to find f'(x)

b) Find f'(5) $= 2(5) = 10$

f'(5) = 10

10) $f(x) = \frac{2}{x}$

Find a formula to find the slope of a tangent line.

11) A toy rocket is launched straight up so that its height *s*, in meters, at time *t*, in seconds, is given by $s(t) = -2t^2 + 20t$.

a) Find s'(t)

11b) Find s'(2) $= -4(2) + 20 = -8 + 20 = 12$

$s'(2) = 12$

11c) Interpret your answer to part b. velocity is 12 meters per second