#1 – 13: Use the appropriate technique to find the derivatives of the following functions.

1) $f(x) = 3x^2 - 5x + 4$

This should be solved using the power rule.

$$f'(x) = \frac{d}{dx}(3x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(4)$$

$$f'(x) = 2 \cdot 3x^1 - 5 + 0$$

$$f'(x) = 6x - 5$$

Answer: $f'(x) = 6x - 5$

2) $f(x) = \frac{-3}{x^2}$

$f(x) = -3x^{-2}$

This can be done by rewriting and then using the power rule,

Or

$f'(x) = -2 \cdot -3x^{-2-1}$

You do not rewrite and use the quotient rule

$f'(x) = 6x^{-3}$

REWRITE AND USE POWER RULE METHOD

$f'(x) = \frac{6}{x^3}$

QUOTIENT RULE METHOD

Denominator x^2	Numerator -3
Derivative $2x$	Derivative Type equation here. 0
cross multiply top down $0 \cdot x^2 = 0$	*cross multiply bottom up* $-3 \cdot 2x = -6x$

$$f'(x) = \frac{0 - (-6x)}{(x^2)^2}$$

$$f'(x) = \frac{6x}{x^4}$$

Answer: $f'(x) = \frac{6}{x^3}$

$$f'(x) = \frac{6x}{xxxx} = \frac{6}{x^3}$$

3) $f(x) = 2\sqrt[3]{x^2}$

This needs to be done by rewriting and then using the power rule,

$$f(x) = 2x^{2/3}$$

$$f'(x) = \frac{2}{3} \cdot 2x^{2/3 - 3/3}$$

$$f'(x) = \frac{4}{3}x^{-1/3}$$

$$f'(x) = \frac{4}{3} \cdot \frac{1}{x^{1/3}}$$

Answer: $f'(x) = \frac{4}{3\sqrt[3]{x}}$

$$f'(x) = \frac{4 \cdot 1}{3 \cdot \sqrt[3]{x}}$$

$$f'(x) = \frac{4}{3\sqrt[3]{x}}$$

4) $f(x) = \frac{5x^2+3}{x^2}$ $\quad f(x) = \frac{5x^2}{x^2} + \frac{3}{x^2}$

This can be done by rewriting and then using the power rule,

Or

You do not rewrite and use the quotient rule

$$f(x) = 5 + 3x^{-2}$$

$$f(x) = 3x^{-2} + 5$$

REWRITE AND USE POWER RULE METHOD

$$f'(x) = -2 \cdot 3x^{-2-1} + 0$$

$$f'(x) = -6x^{-3}$$

$$f'(x) = \frac{-6}{x^3}$$

QUOTIENT RULE METHOD

Denominator x^2	Numerator $5x^2+3$
Derivative $2x$	Derivative Type equation here. $10x$
cross multiply top down $x^2 \cdot 10x = 10x^3$	*cross multiply bottom up* $2x(5x^2+3) = 10x^3+6x$

$$f'(x) = \frac{10x^3 - (10x^3 + 6x)}{(x^2)^2}$$

Answer: $f'(x) = -\frac{6}{x^3}$

$$f'(x) = \frac{10x^3 - 10x^3 - 6x}{x^4} = \frac{-6\cancel{x}}{\cancel{x}xxx}$$

$$f'(x) = \frac{-6}{x^3}$$

5) $f(x) = (x^2 + 6x)(3x - 1)$

This can be done by rewriting and then using the power rule,

Or F $x^2 \cdot 3x$ O $x^2 \cdot -1$ I $6x \cdot 3x$ L $6x \cdot -1$

You do not rewrite and use the Product rule

$3x^3 - 1x^2 + 18x^2 - 6x$

REWRITE AND USE POWER RULE METHOD

$f(x) = 3x^3 + 17x^2 - 6x$

$f'(x) = 3 \cdot 3x^2 + 2 \cdot 17x - 6$

$f'(x) = 9x^2 + 34x - 6$

PRODUCT RULE METHOD

First factor $x^2 + 6x$	Second Factor $3x - 1$
Derivative $2x + 6$	Derivative 3
cross multiply top down $3(x^2 + 6x) = 3x^2 + 18x$	*cross multiply bottom up* $(2x+6)(3x-1)$

$= 6x^2 - 2x + 18x - 6$

$= 6x^2 + 16x - 6$

$f'(x) = 3x^2 + 18x$

$+ 6x^2 + 16x - 6$

$f'(x) = 9x^2 + 34x - 6$

$Answer:\ f'(x) = 9x^2 + 34x - 6$

6) $f(y) = \frac{y^2}{3y-5}$

You must use the quotient rule for this derivative.

Denominator $3y-5$	Numerator y^2
Derivative 3	Derivative Type equation here. $2y$
cross multiply top down $2y(3y-5)$	*cross multiply bottom up* $3y^2$

$6y^2-10y$

$$f'(y) = \frac{6y^2-10y-3y^2}{(3y-5)^2}$$

$$f'(y) = \frac{3y^2-10y}{(3y-5)^2}$$

answer: $f'(y) = \frac{y(3y-10)}{(3y-5)^2}$

$$f'(y) = \frac{y(3y-10)}{(3y-5)^2}$$

7) $f(t) = 2(4t-3)^5$

This is a chain rule problem.

CHAIN RULE short cut to find a derivative of a problem written in the form:

$$f(x) = a(inside\ parenthesis)^n$$

$$f'(x) = n * a * (derivative\ of\ inside\ of\ parenthesis)^{n-1}$$

$a = 2$

$n = 5$

Derivative INSIDE PARENTHESIS 4

$f'(x) = 5 \cdot 2 \cdot 4(4T-3)^4$

Answer: $f'(t) = 40(4t-3)^4$

$f'(x) = 40(4T-3)^4$

8) $y = 4x^3(5x+3)^2$

This is first a product rule problem. You will need to use the chain rule as one derivative in the power rule.

First factor $4x^3$	Second Factor $(5x+2)^2$
Derivative $12x^2$	Derivative $2 \cdot 5(5x+2) = 10(5x+2)$
cross multiply top down $4x^3 \cdot 10(5x+2)$	*cross multiply bottom up* $12x^2(5x+2)^2$

$$y' = 40x^3(5x+3) + 12x^2(5x+3)^2$$

$$= 4(10x^3(5x+3) + 3x^2(5x+3)^2)$$

$$= 4x^2(10x(5x+3) + 3(5x+3)^2)$$

Answer: $y' = 4x^2(5x+3)(25x+9)$

$$= 4x^2(5x+3)(10x + 3(5x+3))$$

$$10x + 15x + 9$$

$$y' = 4x^2(5x+3)(25x+9)$$

9) $f(x) = x^2 + 3x; at\ x = 2$

a) Find the slope of the tangent line to the graph of the function for the given value of x.

b) Find the equation of the tangent line to the graph of the function for the given value of x.

a) $f'(x) = 2x + 3$

$m = f'(2) = 2(2) + 3$

$\boxed{m = 7}$

b) y-coord point

$y = f(2) = (2)^2 + 3(2) = 10$

Point $(2, 10)$ Slope $m = 7$

$y - 10 = 7(x - 2)$

$y - 10 = 7x - 14$

$+10 \quad +10$

$\boxed{y = 7x - 4}$

Answer 9a) $m = 7$ 9b) $y = 7x - 4$

10) $f(x) = x^2 + 8x - 4$

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

a) $f'(x) = 2x + 8$

$2x + 8 = 0$

$2x = -8$

$x = -4$

b) y coord point

$y = f(-4) = (-4)^2 + 8(-4) - 4$

$= -20$

Point $(-4, -20)$

Slope $m = 0$

All horizontal lines have $m = 0$

$y - (-20) = 0(x - (-4))$

$y + 20 = 0$

$-20 \quad -20$

$\boxed{y = -20}$

10a) $x = -4$ 10b) $y = \cancel{-16}$ -20

Chapter 2 Practice Test Part 2

11) $f(x) = e^{x^2}$

Rule needed $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$c = 1$

$g(x) = x^2$

$g'(x) = 2x$

$f'(x) = 1 \cdot 2xe^{x^2}$

Answer: $f'(x) = 2xe^{x^2}$

$f'(x) = 2xe^{x^2}$

12) $f(y) = (2y-4)e^{5y^2}$

This is a product rule problem. We will need to find an "e" derivative during the product rule.

Rule needed for the "e" $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor $2y-4$	Second Factor e^{5y^2}
Derivative 2	Derivative $10ye^{5y^2}$
cross multiply top down $(2y-4)10ye^{5y^2}$	*cross multiply bottom up* $2e^{5y^2}$

$$f'(y) = 10y(2y-4)e^{5y^2} + 2e^{5y^2}$$

$$f'(y) = 2e^{5y^2}(5y(2y-4)+1)$$

$10y^2 - 20y$

$$f'(y) = 2e^{5y^2}(10y^2-20y+1)$$

Answer: $f'(y) = 2e^{5y^2}(10y^2 - 20y + 1)$

13) $f(t) = \frac{t^4}{e^t}$

This is a product rule problem. We will need to find an "e" derivative during the product rule.

Rule needed for the "e"
$f(x) = ce^{g(x)}$
$f'(x) = cg'(x)e^{g(x)}$
Where "c" is a constant (number without a letter)

Also need the quotient rule because of the division.

Denominator e^T	Numerator T^4
Derivative e^T	Derivative $4T^3$
cross multiply top down $4T^3e^T$	*cross multiply bottom up* T^4e^T

$$f'(T) = \frac{4T^3e^T - T^4e^T}{(e^T)^2} = \frac{T^3\cancel{e^T}(4-T)}{e^T\cancel{e^T}}$$

$$f'(T) = \frac{T^3(4-T)}{e^T}$$

Answer: $f'(t) = \frac{t^3(-t+4)}{e^t} = \frac{-t^3(t-4)}{e^t}$

$$\text{or } f'(T) = \frac{-T^3(T-4)}{e^T}$$

14) $f(t) = \ln(3t^5)$

Rule needed $f(x) = c\ln[g(x)]$ $f'(x) = \frac{cg'(x)}{g(x)}$ c *is a constant*

$c = 1$

$g(T) = 3T^5$

$g'(T) = 15T^4$

$f'(T) = \frac{1 \cdot 15T^4}{3T^5}$

Answer: $f'(t) = \frac{5}{t}$

$f'(T) = \frac{\overset{5}{\cancel{15}}\,\cancel{T}\cancel{T}\cancel{T}\cancel{T}}{\underset{1}{\cancel{3}}\,\cancel{T}\cancel{T}\cancel{T}\cancel{T}T}$

$f'(T) = \frac{5}{T}$

15) $y = x^2 ln(x)$

This is a product rule problem. We will need to find an “ln” derivative during the product rule.

Rule needed for ln $f(x) = cln[g(x)]$ $f'(x) = \frac{cg\prime(x)}{g(x)}$ $c \text{ is a constant}$

Also need product rule

First factor x^2	Second Factor $\ln(x)$
Derivative $2x$	Derivative $1/x$
cross multiply top down $x^2 \cdot \frac{1}{x} = x$	*cross multiply bottom up* $2x \ln(x)$

$$y' = x + 2x\ln(x)$$

$$y' = x(1 + 2\ln(x))$$

Answer: $\frac{dy}{dx} = x + 2xln(x) \;\; or \;\; x(1 + 2\ln(x)) \; or \;\; x(2\ln(x) + 1)$

16) $f(x) = e^{x^2}$

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

a) Find derivative, then solve derivative equal to zero.

Rule needed for the derivative
$f(x) = ce^{g(x)}$
$f'(x) = cg'(x)e^{g(x)}$
Where "c" is a constant (number without a letter)

a) $f(x) = e^{x^2}$

$f'(x) = 2xe^{x^2}$

$2xe^{x^2} = 0$

$2x = 0 \qquad e^{x^2} = 0$

$\frac{2x}{2} = \frac{0}{2} \qquad$ NO Sol

$x = 0$

ANSWER PART A $\boxed{x = 0}$

16a) $x = 0$ 16b) $y = 1$

b) y coord point $y = f(0) = e^{0^2} = e^0 = 1$

point $(0, 1)$ $m = 0$ All horizontal lines have slope 0

$y - 1 = 0(x - 0)$

$y - 1 = 0$

$y = 1$

ANSWER PART b $y = 1$

17) Suppose that the cost in dollars to make x super-sized candy bars is given by: $C(x) = \ln(x) + 0.15x$

a) Find C(4) (round to 2-decimals)

$C(4) = Ln(4) + 0.15(4) = 1.986$

$C(4) = 1.99$

b) Interpret your answer to part a.

See below

c) Create the marginal cost function $C'(x)$ for this product.

$C'(x) = \frac{1}{x} + 0.15$

d) Find C'(4) (round to 2 decimals)

$C'(4) = \frac{1}{4} + 0.15 = 0.40$

e) Interpret your answer to question part d.

See below

Answers: 17a) $C(4) = 1.99$

17b) It will cost $1.99 to make 4 super-sized candy bars.

17c) $C'(x) = \frac{1}{x} + .15$

17d) $C'(4) = 0.40$

17e) It will cost $0.40 or 40 cents to make the 5th candy bar.

18) A Corporation determines the weekly profit (P(x)) from selling certain widget in produces and sells:

$P(x) = -0.01x^2 + 20x - 2000 \quad 0 \leq x \leq 1000.$

a) Find P(500)
b) Interpret your answer to part a. (round your answer to 2 decimals)
c) Create the marginal profit function $P'(x)$ for this product.
d) Find P'(500).
e) Interpret your answer to part d.

a) $P(500) = -0.01(500)^2 + 20(500) - 2000$
$= 5500$

b) See below

c) $P'(x) = 2 \cdot -0.01X + 20$
$P'(x) = -0.02X + 20$

d) $P'(500) = -0.02(500) + 20$
$= 10$

e) See below

Answers:
18a) $P(500) = 5500$
18b) Profit will be \$5500 in a week when 500 widgets are produced and sold.
18c) $P'(x) = -0.02x + 20$
18d) $P'(500) = 10$
18e) Company will earn an additional \$10 in profit when 501st widget is produced and sold.