Chapter 3 Practice Test Part 1 (Complete all problems)

1) Consider the graph of the function f(x) below.

1a) interval(s) where the graph is increasing. $(-\infty, -1) \cup (0,1)$
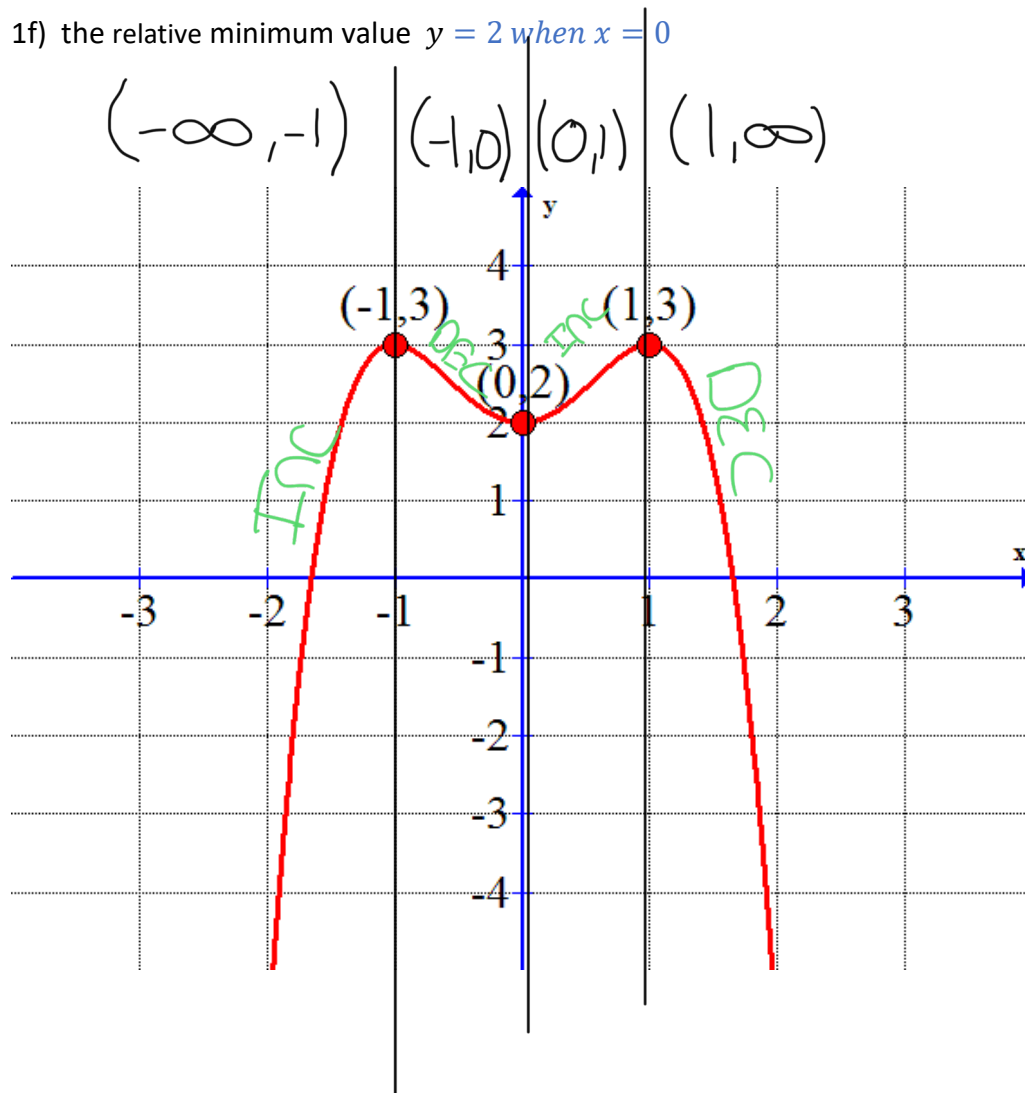
1b) interval(s) where the graph is decreasing. $(-1,0) \cup (1, \infty)$

1c) the coordinates of relative maximum point if any $(-1,3)$ and $(1,3)$

1d) the relative maximum value $y = 3$ when $x = -1, 1$

1e) the coordinates of the relative minimum point if any $(0,2)$

1f) the relative minimum value $y = 2$ when $x = 0$

$$\left(-\infty, -1\right) \quad \left(-1,0\right) \left(0,1\right) \quad \left(1, \infty\right)$$

2) $f(x) = 6xe^x$

Find the following:

2a) f'(x) $f'(x) = 6e^x(x+1)$

2b) the critical numbers $x = -1$

2c) interval(s) where the graph is increasing. $(-1, \infty)$

2d) interval(s) where the graph is decreasing. $(-\infty, -1)$

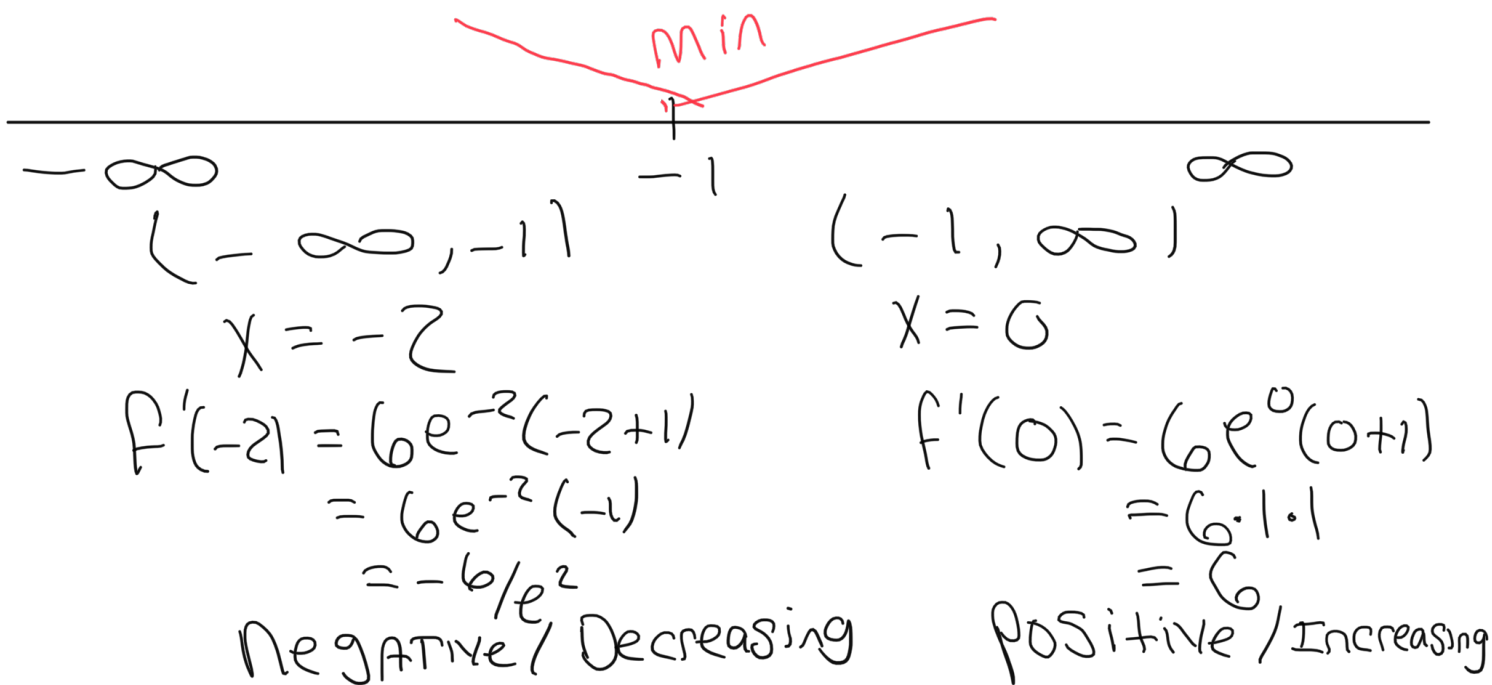2e) the coordinates of relative maximum point if any none

2f) the relative maximum value none

2g) the coordinates of the relative minimum point if any $\left(-1, \frac{-6}{e}\right)$

2h) the relative minimum value $y = -\frac{6}{e}$ when $x = -1$

2a) 1ST $6x$ 2ND e^x $f'(x) = 6xe^x + 6e^x$
    deriv 6   deriv e^x $f'(x) = 6e^x(x+1)$

2b) $6e^x = 0$ $x+1 = 0$ $x = -1$
    No Solution $x = -1$

min

$-\infty$ -1 ∞

$(-\infty, -1)$ $(-1, \infty)$

$x = -2$ $x = 0$

$f'(-2) = 6e^{-2}(-2+1)$ $f'(0) = 6e^0(0+1)$
$= 6e^{-2}(-1)$ $= 6 \cdot 1 \cdot 1$
$= -6/e^2$ $= 6$
Negative / Decreasing   Positive / Increasing

No Max

Min $x = -1$
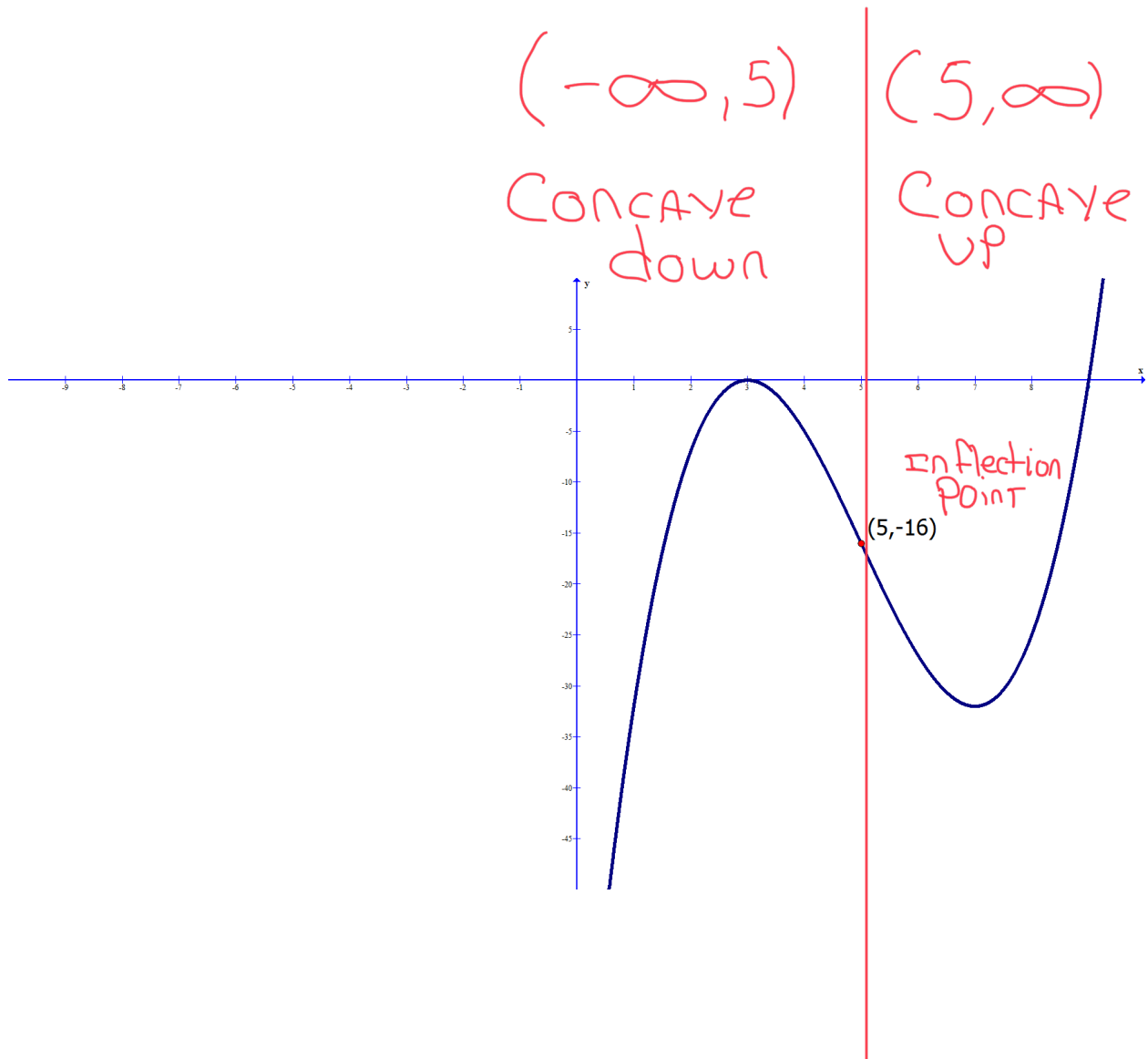$y = f(-1) = 6(-1)e^{-1} = -6e^{-1} = \frac{-6}{e}$

Min $\left(-1, -6/e\right)$

3) Consider the graph of the function f(x) below.

3a) Find the open interval(s) where the graph of the function is concave up $(5, \infty)$
3b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, 5)$
3c) Find all inflection points $(5, -16)$

$(-\infty, 5)$ $(5, \infty)$

CONCAVE down   CONCAVE up

Inflection Point

(5,-16)

4) $f(x) = x^3 - 30x^2$

4a) Find the open interval(s) where the graph of the function is concave up

4b) Find the open interval(s) where the graph of the function is concave down.
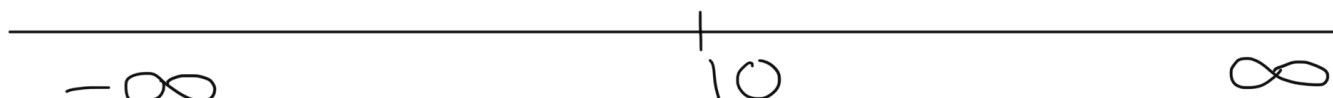
4c) Find all inflection points

$$f'(x) = 3x^2 - 60x$$
$$f''(x) = 6x - 60$$
$$6x - 60 = 0$$
$$6x = 60$$
$$x = 10$$

―――――――――――――――――――――

$-\infty$ _____ 10 _____ ∞

$(-\infty, 10)$ $(10, \infty)$

$x = 0$ $x = 11$

$f''(0) = 6(0) - 60$ $f''(11) = 6(11) - 60$

$\quad = -60$ $\quad = 6$

Concave Down   Concave up

Inflection point $x = 10$

$$y = f(10) = (10)^3 - 30(10)^2 = -2000$$

4a) Find the open interval(s) where the graph of the function is concave up $(10, \infty)$

4b) Find the open interval(s) where the graph of the function is concave down. $(-\infty, 10)$

4c) Find all inflection points $(10, -2000)$

Inflection Point
$(10, -2000)$

Chapter 3 Practice Test Part 2

5) $f(x) = x^3 - 3x$

Find the following.

a) Find the x-intercept(s) if any

$$x^3 - 3x = 0$$
$$x(x^2 - 3) = 0$$
$$x = 0 \qquad x^2 - 3 = 0$$
$$\sqrt{x^2} = \pm\sqrt{3}$$
$$x = \pm\sqrt{3}$$

b) Find the y-intercept, in there is one

$$y = f(0) = (0)^3 - 3(0) = 0$$

$\boxed{\text{y-intercept } (0,0)}$

$\boxed{\begin{array}{l} \text{X-intercepts} \\ (0,0) \ (\sqrt{3},0) \ (-\sqrt{3},0) \end{array}}$

c) Find the interval(s) where the graph of the function is increasing
d) Find the interval(s) where the graph of the function is decreasing
e) Find all relative maxima
f) Find all relative minima

$$f(x) = x^3 - 3x$$
$$f'(x) = 3x^2 - 3$$

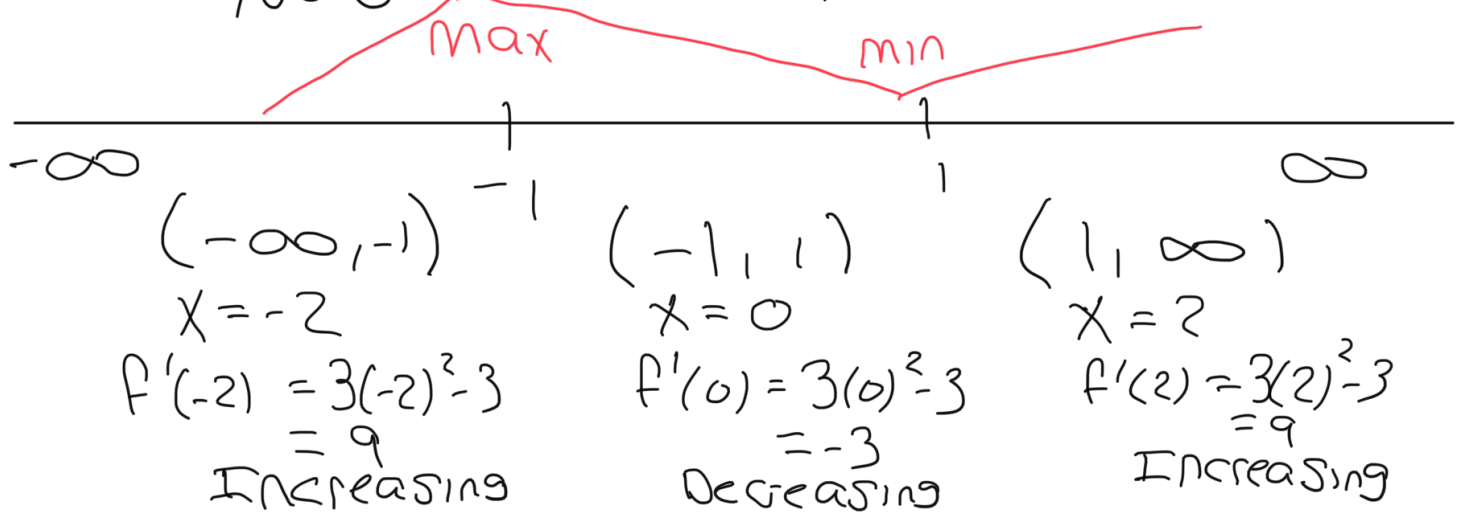$$3x^2 - 3 = 0$$
$$3(x^2 - 1) = 0$$
$$3(x+1)(x-1) = 0$$

$$3 = 0$$
NO SOLUTION

$$x + 1 = 0$$
$$x = -1$$

$$x - 1 = 0$$
$$x = 1$$

$(-\infty, -1)$
$x = -2$
$f'(-2) = 3(-2)^2 - 3$
$= 9$
Increasing

$(-1, 1)$
$x = 0$
$f'(0) = 3(0)^2 - 3$
$= -3$
Decreasing

$(1, \infty)$
$x = 2$
$f'(2) = 3(2)^2 - 3$
$= 9$
Increasing

Max $x = -1$
$y = f(-1) = (-1)^3 - 3(-1) = 2$

Min $x = 1$
$y = f(1) = (1)^3 - 3(1) = -2$

Max
$(-1, 2)$

Min
$(1, -2)$

g) Find the interval(s) where the graph of the function is concave up (if any)
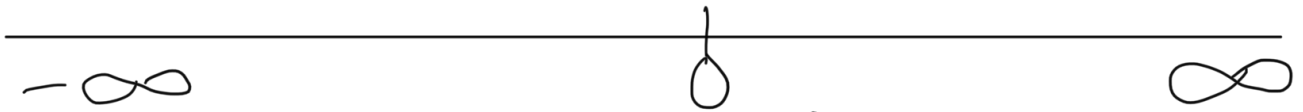h) Find the interval(s) where the graph of the function is concave down (if any)
i) Find all inflection points (if any)

$$f(x) = x^3 - 3x$$
$$f'(x) = 3x^2 - 3$$
$$f''(x) = 6x$$

$$\frac{6x = 0}{6} \quad \frac{}{6}$$
$$x = 0$$

$-\infty$ ——————————○—————————— ∞

$(-\infty, 0)$ $(0, \infty)$

$x = -1$ $x = 1$

$f''(-1) = 6(-1) = -6$ $f''(1) = 6(1) = 6$
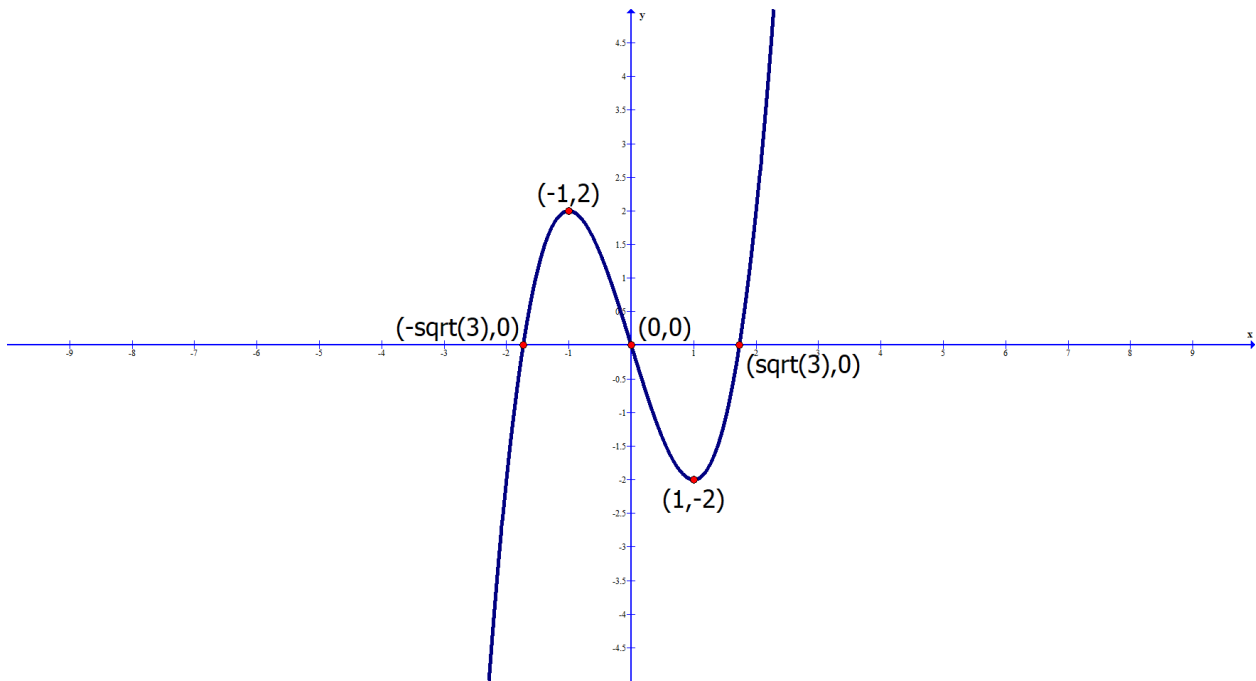
Concave Down   Concave up

Inflection Point

$x = 0$
$$y = f(0) = (0)^3 - 3(0) = 0$$
$$I.P. \ (0, 0)$$

j) Sketch a graph

5a) Find the x-intercept(s), if any $(0,0)\ and\ (\sqrt{3},0)\ (-\sqrt{3},0)$
5b) Find the y-intercept, in there is one $(0,0)$
5c) Find the interval(s) where the graph of the function is increasing $(-\infty,-1)\cup(1,\infty)$
5d) Find the interval(s) where the graph of the function is decreasing $(-1,1)$
5e) Find all relative maxima $(-1,2)$
5f) Find all relative minima $(1,-2)$
5g) Find the interval(s) where the graph of the function is concave up (if any) $(0,\infty)$
5h) Find the interval(s) where the graph of the function is concave down (if any) $(-\infty,0)$
5i) Find all inflection points (if any) $(0,0)$
5j) Sketch a graph

6): Find the following. $f(x) = \frac{x-2}{x+3}$

hint: $f'(x) = \frac{5}{(x+3)^2}$ $f''(x) = \frac{-10}{(x+3)^3}$

a) Find the domain

$X + 3 = 0$

$X = -3$

All REal numbers Except $X = -3$

OR $(-\infty, -3) \cup (-3, \infty)$

b) Find the equation of the vertical asymptote

$X = -3$

c) Find the x-intercept(s), if any

$X - 2 = 0$

$X = 2$

$(2, 0)$

d) Find the y-intercept, in there is one

$y = f(0) = \frac{0-2}{0+3} = \frac{-2}{3}$ $(0, -2/3)$

e) Find all horizontal asymptotes

$\frac{1 \, X - 2}{1 \, X + 3}$

$y = \frac{1}{1}$

$y = 1$

f) Find the interval(s) where the graph of the function is increasing
g) Find the interval(s) where the graph of the function is decreasing
h) Find all relative maxima and
i) Find all relative minima

$$f'(x) = \frac{5}{(x+3)^2}$$
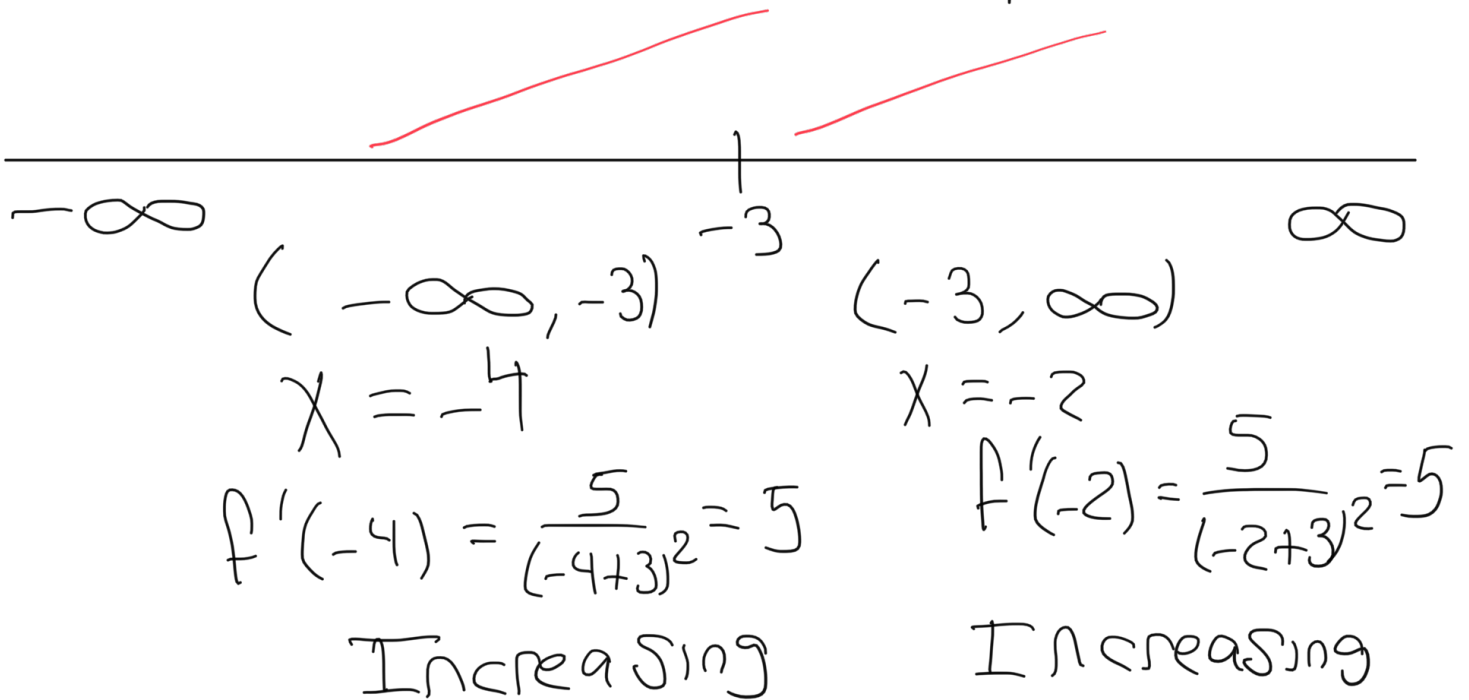
$$5 = 0$$
NO SolUTion

$$(x+3)^2 = 0$$
$$x = -3$$

-3

$$(-\infty, -3)$$
$$x = -4$$
$$f'(-4) = \frac{5}{(-4+3)^2} = 5$$
Increasing

$$(-3, \infty)$$
$$x = -2$$
$$f'(-2) = \frac{5}{(-2+3)^2} = 5$$
Increasing

Never decreasing
no max
no min

j) Find the interval(s) where the graph of the function is concave up (if any)
k) Find the interval(s) where the graph of the function is concave down (if any)
l) Find all inflection points (if any)

$$f''(x) = \frac{-10}{(x+3)^3}$$

$$-10 = 0$$
No Solution

$$(x+3)^3 = 0$$
$$x = -3$$

$$-\infty \qquad\qquad -3 \qquad\qquad \infty$$

$$(-\infty, -3) \qquad (-3, \infty)$$

$$x = -4 \qquad\qquad x = -2$$

$$f''(-4) = \frac{-10}{(-4+3)^3} = 10 \qquad f''(-2) = \frac{-10}{(-2+3)^3} = -10$$

Concave up   Concave Down

No Inflection point at $x = -3$
Since $x = -3$ is not in the
Domain

m) Sketch a graph

6a) Find the domain $(-\infty, -3) \cup (-3, \infty)$

6b) Find the equation of the vertical asymptote $x = -3$

6c) Find the x-intercept(s), if any $(2,0)$

6d) Find the y-intercept, in there is one $\left(0, -\frac{2}{3}\right)$

6e) Find all horizontal asymptotes $y = 1$

6f) Find the interval(s) where the graph of the function is increasing $(-\infty, -3) \cup (-3, \infty)$

6g) Find the interval(s) where the graph of the function is decreasing never
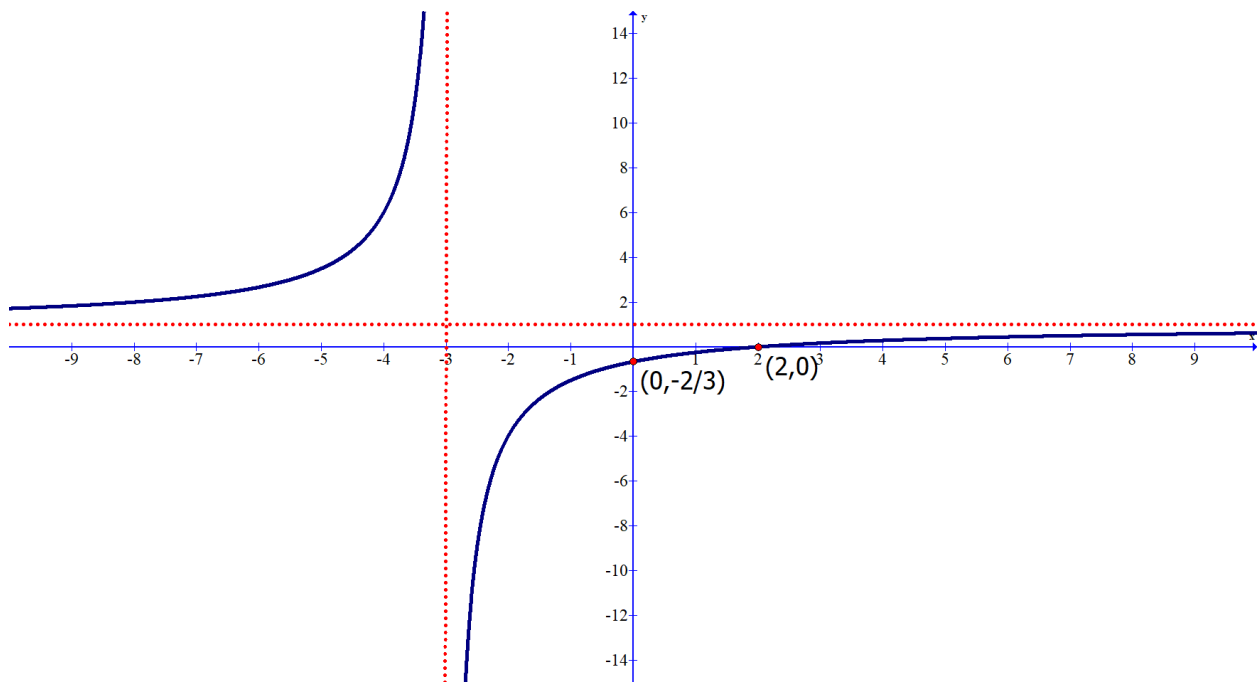
6h) Find all relative maxima none

6i) Find all relative minima none

6j) Find the interval(s) where the graph of the function is concave up (if any) $(-\infty, -3)$

6k) Find the interval(s) where the graph of the function is concave down (if any) $(-3, \infty)$

6l) Find all inflection points (if any) none

6m) Sketch a graph

7) A company makes a single product. The cost function for the product is given by:
$C(x) = 0.5x^2 + 30x + 100$ where $C(x)$ is the total cost to produce x units of the product.
The demand function is given by $p(x) = -2x + 90$, where $p(x)$ is the price to sell x units of the product.

7a) Create a revenue function. $R(x) = x \cdot p(x) = x(-2x + 90)$

$$\boxed{R(x) = -2x^2 + 90x}$$

7b) Create a profit function.
$P(x) = -2x^2 + 90x - (0.5x^2 + 30x + 100)$
$P(x) = -2x^2 + 90x - 0.5x^2 - 30x - 100$

$$\boxed{P(x) = -2.5x^2 + 60x - 100}$$

7c) How many units must the company produce and sell to maximize profit?

$$P'(x) = 2(-2.5)x + 60$$
$$P'(x) = -5x + 60$$
$$-5x + 60 = 0$$
$$-5x = -60$$
$$x = 12$$

$$\boxed{12 \text{ units}}$$

Should confirm this is an x for a max

7d) What is the maximum profit?

$P(12) = -2.5(12)^2 + 60(12) - 100 = 260$

$$\boxed{\$260}$$

7e) What price per unit must be charged to make maximum profit?

$$p(x) = -2x + 90$$
$$p(12) = -2(12) + 90$$
$$= -24 + 90$$
$$= 66$$

$$\boxed{\$66}$$

7a) Create a revenue function. $R(x) = -2x^2 + 90x$
7b) Create a profit function. $P(x) = $ ~~$-2.5x^2 + 60x - 100$~~ $-2.5x^2 + 60x - 100$
7c) How many units must the company produce and sell to maximize profit? ~~~~ 12 units
7d) What is the maximum profit? ~~~~ $\$260$
7e) What price per unit must be charged to make maximum profit? ~~~~ $\$66$