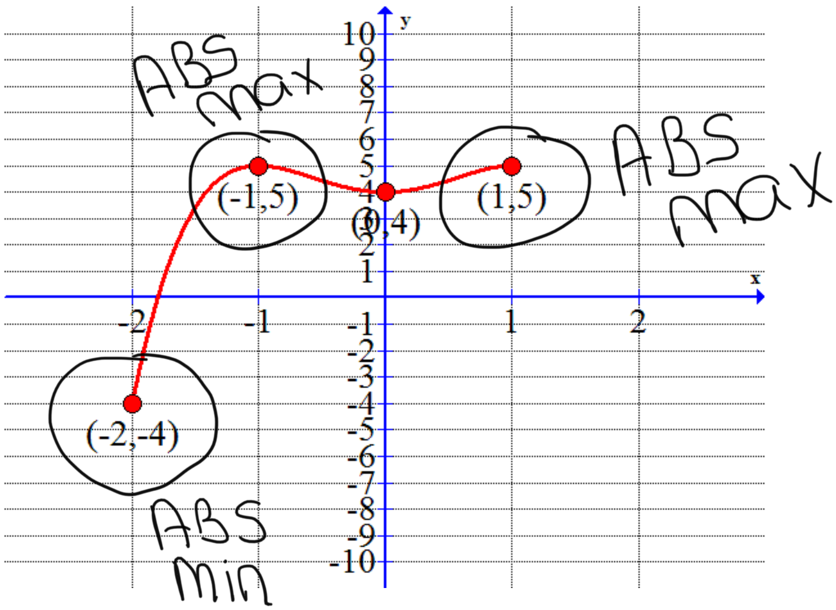
Chapter 4 Practice Test (Complete all problems)

1) Find the absolute maximum and absolute minimum



Answer:

Absolute max of y = 5 which occurs when x = 1 and x = -1,

Absolute min of y = -4 when x = -2

#2-3: Find the absolute maximum and absolute minimum of the function under the given interval.

2) $f(x) = 2x^3 - 54x$; [0,4]

$$f'(x) = 6x^2 - 54$$
$$6x^2 - 54 = 0$$
$$6(x^2 - 9) = 0$$
$$6(x+3)(x-3) = 0$$

$6 = 0$ NO Solution

$x + 3 = 0$
$x = -3$
Ignore not between 0, 4

$x - 3 = 0$
$x = 3$

$f(0) = 2(0)^3 - 54(0)$
$= 0$

$f(3) = 2(3)^3 - 54(3)$
$= -108$

$f(4) = 2(4)^3 - 54(4)$
$= -88$

x	$y = f(x)$	
0	0	ABS max
3	-108	ABS min
4	-88	

Answer:

Absolute max of y = 0 when x = 0,

Absolute min of y = -108 when x = 3

#2-3: Find the absolute maximum and absolute minimum of the function under the given interval.

3) $f(x) = xe^x$; [-2,2]

1st x 2nd e^x

Deriv 1 Deriv e^x

$$f'(x) = xe^x + 1e^x$$

$$f'(x) = e^x(x+1)$$

$$e^x(x+1) = 0$$

$e^x = 0$ $\quad$ $x+1=0$

No Solution $\quad$ $x=-1$

$f(-2) = -2e^{-2} = \frac{-2}{e^2} \approx -0.27$

$f(-1) = -1e^{-1} = -1/e \approx -0.37$

$f(2) = 2e^2 \approx 14.78$

-2	$-2/e^2$	
-1	$-1/e$	ABS min
2	$2e^2$	ABS max

Answer:

Absolute max of y = $2e^2$ when x = 2,

Absolute min of y = -1/e when x = -1

4) A campground owner has 100 meters of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river, and let W represent the width of the field and L represent the length of the field. Make W be the side of the fence that is perpendicular to the river so that two widths and one length will need to be constructed.

a) Write an equation for the length of the field

$$L + 2W = 100$$
$$-2W \quad -2W$$

$$\boxed{L = -2W + 100}$$

b) Write an equation for the area of the field.

$$A = LW = (-2W + 100)W$$

$$\boxed{A = -2W^2 + 100W}$$

c) Find the domain of the area equation that was created in part b.
(This domain will be of the form: $\# \leq W \leq \#$)

$$100/2 = 50$$

$$\boxed{0 \leq W \leq 50}$$

d) Find the value of w leading to the maximum area

$$A' = -4W + 100$$

$$-4W + 100 = 0$$
$$-4W = -100$$
$$W = 25$$

W	A
0	0
25	1250
50	0

$-2(25)^2 + 100(25)$

$\boxed{\text{WIDTH } 25\text{m}}$

e) Find the value of L leading to the maximum area

$$L = -2(25) + 100$$
$$L = 50$$

$\boxed{\text{length } 50\text{m}}$

f) Find the maximum area.

$$A = (50)(25)$$

$\boxed{\text{Area } 1250\text{m}^2}$

Answers: 4a) L = $-2W+100$ 4b) A = $-2W^2 + 100W$ 4c) Domain $0 \leq W \leq 50$

4d) W = 25 m 4e) L = 50 m 4f) Area = 1250 m^2

#5-6: Use implicit differentiation to determine $\frac{dy}{dx}$.

5) $y^2 - 3y = 5x^3$

$$\frac{d}{dx}(y^2) - \frac{d}{dx}(3y) = \frac{d}{dx}(5x^3)$$

$$2y\frac{dy}{dx} - 3\frac{dy}{dx} = 15x^2$$

$$\frac{dy}{dx}\frac{(2y-3)}{2y-3} = \frac{15x^2}{2y-3}$$

Answer: $\frac{dy}{dx} = \frac{15x^2}{2y-3}$

$$\frac{dy}{dx} = \frac{15x^2}{2y-3}$$

#5-6: Use implicit differentiation to determine $\frac{dy}{dx}$.

6) $xy = 6x + 8$

$$\frac{d}{dx}(xy) = \frac{d}{dx}(6x) + \frac{d}{dx}(8)$$

$$x\frac{dy}{dx} + y = 6$$

$$-y \quad -y$$

$$\frac{x\frac{dy}{dx}}{x} = \frac{-y+6}{x}$$

$$dy/dx = \frac{-y+6}{x}$$

$$\frac{d}{dx}(xy)$$

1ST x   last y

DERIV 1   deriv 1 $\frac{dy}{dx}$

$$x\frac{dy}{dx} + 1y$$

$$x\frac{dy}{dx} + y$$

Answer: $\frac{dy}{dx} = \frac{-y+6}{x} = \frac{-(y-6)}{x}$

7) Find the equation of the line tangent to the graph at the indicated point.

$y^2 = 3y + 2x^2 - 2$; (1,3)

(We will need to find $\frac{dy}{dx} = \frac{4x}{2y-3}$ first)

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(3y) + \frac{d}{dx}(2x^2) - \frac{d}{dx}(2)$$

$$2y\frac{dy}{dx} = 3\frac{dy}{dx} + 4x - 0$$

$$-3\frac{dy}{dx} \quad -3\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - 3\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx}\frac{(2y-3)}{2y-3} = \frac{4x}{2y-3}$$

$$dy/dx = \frac{4x}{2y-3}$$

$$m = \frac{4(1)}{2(3)-3} = \frac{4}{3}$$

Answer: $y = \frac{4}{3}x + \frac{5}{3}$

$$y - 3 = \frac{4}{3}(x-1)$$

$$y - \frac{3}{1}\frac{(3)}{(3)} = \frac{4}{3}x - \frac{4}{3}$$

$$y - 9/3 = \frac{4}{3}x - 4/3$$

$$+9/3 \quad +9/3$$

$$y = 4/3x + 5/3$$

8) A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius *r* of the outer ripple is increasing at a constant rate of 3 feet per second. When the radius is 6 feet, at what rate is the total area of the disturbed water changing?

1. Identify the information that is given.

$$\frac{dr}{dT} = 3 \text{ FT/SEC}$$
$$r = 6$$

2. Identify what needs to be solved for.

Find $\frac{dA}{dT}$

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

$$A = \pi r^2$$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$$\frac{dA}{dT} = 2\pi r \frac{dr}{dT}$$

$$\frac{d}{dT}(A) = \frac{d}{dT}(\pi r^2)$$

5. Solve for the unknown rate of change.

Done with last STEP
(No Extra Algebra)

$$\frac{dA}{dT} = 2\pi r \frac{dr}{dT}$$

6. Substitute all known values to get the final answer.

$$\frac{dA}{dt} = 2\pi(6)(3)$$
$$= 18\pi$$

Answer: Area is growing at rate of $36\pi\ ft^2/sec$

9) Air is being pumped into a spherical balloon at 3 cm^3/second. Calculate the rate at which the radius of the balloon is changing when the radius of the balloon is 10 cm.

1. Identify the information that is given.

$$\frac{dV}{dt} = 3\ cm^3/sec$$

$$r = 10$$

2. Identify what needs to be solved for.

Find dr/dt

3. Write an equation that relates the variables. To derive the equation you may use a geometric fact (like an area or a volume formula)

$$\boxed{V = \frac{4}{3}\pi r^3}$$

4. Take the derivative $\frac{d}{dt}$ of both sides of the equation.

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\boxed{\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}}$$

$$\underline{}\ 4\pi r^2 \qquad \underline{}\ 4\pi r^2$$

5. Solve for the unknown rate of change.

$$dr/dt = \frac{dV/dt}{4\pi r^2}$$

6. Substitute all known values to get the final answer.

$$\frac{dr}{dt} = \frac{3}{4\pi(10)^2}$$

$$\frac{dr}{dt} = \frac{3}{4\pi(100)}$$

$$\frac{dr}{dt} = \frac{3}{400\pi}$$

Answer: Radius is growing at a rate of $\frac{3}{400\pi}\ cm/sec$