1. Find the antiderivatives for each of the following using any method you prefer:

A) \[ \int x \sec^2(x) \, dx \]

By parts:

\[ u = x \]
\[ du = dx \]
\[ dv = \sec^2(x) \, dx \]
\[ v = \tan(x) \]

\[ x \tan(x) + \ln | \cos(x) | + C \]

B) \[ \int \sin^2(3x) \, dx \]

\[ \frac{1}{2} \int \cos(6x) \, dx - \frac{1}{2} \int \cos(6x) \, dx \]
\[ u = 6x \]
\[ du = 6 \, dx \]
\[ \frac{1}{2} \cdot \frac{1}{6} \int \cos(6x) \, dx + C \]

C) \[ \int \cos^2(x) \sin^3(x) \, dx \]

\[ v = \cos(x) \]
\[ dv = -\sin(x) \, dx \]
\[ du = \sin^2(x) \, dx \]
\[ \int [ \cos^2(x) \sin^3(x) ] \, dx \]
\[ \frac{1}{3} \cos^3(x) + \frac{1}{3} \cos^5(x) + C \]

D) \[ \int \sec^3(x) \tan(x) \, dx \]

\[ u = \sec(x) \]
\[ dv = \tan(x) \, dx \]
\[ v = \sec(x) \]
\[ \frac{1}{3} \sec^3(x) + C \]

E) \[ \int \sin(5x) \cos(2x) \, dx \]

\[ u = 5x \]
\[ du = 5 \, dx \]
\[ \frac{1}{2} \int \sin(3x) \, dx + \frac{1}{2} \int \sin(7x) \, dx \]
\[ \frac{1}{6} \cos(3x) - \frac{1}{14} \cos(7x) + C \]

G) \[ \int \frac{1}{\sqrt{9x^2 + 25}} \, dx \]

\[ u = 3x \]
\[ dv = 3 \, dx \]
\[ a = 5 \]
\[ \frac{1}{3} \int \frac{1}{\sqrt{u^2 + a^2}} \, du \]
\[ \frac{1}{3} \ln | 3x + \sqrt{9x^2 + 25} | + C \]
1. Continue to find the antiderivatives for each of the following using any method you prefer:

a) \[ \int \frac{1 - 2x}{49 - x^2} \, dx \]

\[ \frac{1}{9} \ln \left| \frac{7+x}{7-x} \right| + \frac{1}{9} \ln |9-x^2| + C \]

b) \[ \int \frac{\sin(x)}{\cos^2(x) + 1} \, dx \]

\[ u = \cos(x) \]
\[ du = -\sin(x) \, dx \]
\[ \int \frac{1}{\sqrt{u^2 + 1}} \, du \]
\[ \ln \left| \cos(x) + \sqrt{\cos^2(x) + 1} \right| + C \]

2. DRAW ONLY a representative right triangle you would use to help you set up the trig substitution routine for the integral: \[ \int \frac{1}{x^2 \sqrt{9x^2 - 4}} \, dx \]. YOU DO NOT NEED TO DO THE TRIG SUB...ONLY DRAW THE right triangle.

3. Use trig substitution to help you do the following integral: \[ \int \frac{1}{\sqrt{x^2 + 4}} \, dx \]

\[ \frac{x}{\sqrt{x^2 + 4}} \]

\[ x = 2 \tan \theta \]
\[ dx = 2 \sec^2 \theta \, d\theta \]
\[ \sqrt{x^2 + 4} = 2 \sec \theta \]

\[ \int \frac{2 \sec^2 \theta \, d\theta}{2 \sec \theta} \]

\[ \int \sec \theta \, d\theta \]

\[ \ln |\sec \theta + \tan \theta| + C \]

\[ \ln \left| \frac{\sqrt{x^2 + 4}}{2} + \frac{x}{2} \right| + C \]
4. Prove the following integration table:

\[
\int \frac{a^2 - u^2}{\sqrt{a^2 - u^2}} \, du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C
\]

\[
\int \frac{\cos^2 \theta}{\sin \theta} \, d\theta = \int \frac{1 - \sin^2 \theta}{\sin \theta} \, d\theta = \int \frac{1}{\sin \theta} \, d\theta - \int \sin \theta \, d\theta
\]

\[
a \int \frac{1}{\sin \theta} \, d\theta = -a \int \csc \theta \, d\theta
\]

\[
a \int \csc \theta \, d\theta = -a \ln \left| \csc \theta + \cot \theta \right| + a \cot \theta + C
\]

\[
a \int \sin \theta \, d\theta = -a \ln \left| \frac{a^2 - u^2}{a} \right| = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C
\]

5. For the integral \( \int \frac{x^2 - x - 6}{x(x-3)^2(x^2+4)} \, dx \), write out the Partial Fraction Decomposition you would use for the integrand. YOU DO NOT NEED TO FIND THE CONSTANTS. SET UP ONLY the partial fraction decomposition.

\[
\frac{x^2 - x - 6}{x(x-3)^2(x^2+4)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \frac{Dx+E}{(x^2+4)}
\]

6. Use Partial Fraction Decomposition to find the following integral:

\[
\int \frac{1}{x^2 - 25} \, dx
\]

\[
\frac{1}{(x-5)(x+5)} = \frac{A}{x-5} + \frac{B}{x+5}
\]

\[
x = 5 \quad 1 = 10A \quad A = \frac{1}{10}
\]

\[
x = -5 \quad 1 = -10B \quad B = \frac{1}{10}
\]

EXTRA CREDIT: Find \( \int e^x \cos(x) \, dx \)

\[
e^{x} \sin(x) - \int e^{x} \cos(x) \, dx
\]

\[
e^{x} \cos(x) + \int e^{x} \sin(x) \, dx
\]

\[
\frac{\int e^{x} \sin(x) \, dx - \int e^{x} \cos(x) \, dx}{2} = \int e^{x} \sin(x) \, dx = \frac{e^{x} \sin(x) + e^{x} \cos(x)}{2}
\]