1. Are the following points the coordinates of the vertices of a right triangle? \((-4, 5) (6, 1) (-8, -5)\)

To answer this question we are going to use the distance formula to find the distance between each pair of points and then see if those distances satisfy the Pythagorean Theorem.

\((-4, 5) (6, 1) (-8, -5)\)

\[d\left((-4, 5) (6, 1)\right) = \sqrt{(6 - (-4))^2 + (1 - 5)^2} = \sqrt{10^2 + (-4)^2} = \sqrt{100 + 16} = \sqrt{116}\]

\[d\left((-4, 5) (-8, -5)\right) = \sqrt{(-8 - (-4))^2 + (-5 - 5)^2} = \sqrt{(-4)^2 + (-10)^2} = \sqrt{16 + 100} = \sqrt{116}\]

\[d\left((6, 1) (-8, -5)\right) = \sqrt{(-8 - 6)^2 + (-5 - 5)^2} = \sqrt{(-14)^2 + (-6)^2} = \sqrt{196 + 36} = \sqrt{232}\]

So does \(a^2 + b^2 = c^2\)? Where "c" is the longest side?

\[\left(\sqrt{116}\right)^2 + \left(\sqrt{116}\right)^2 = \left(\sqrt{232}\right)^2 \rightarrow \sqrt{116} + \sqrt{116} = 232 \quad YES!!!!\]

So the given coordinates are the vertices of a right triangle!!!

2. Find the center and radius of each circle.

A. \((x - 1)^2 + (y - 3)^2 = 16\) Center = \((1, 3)\) Radius = 4

B. \((x + 3)^2 + y^2 = 25\) Center = \((-3, 0)\) Radius = 5

C. \(x^2 + (y + 2)^2 = 1\) Center = \((0, -2)\) Radius = 1

3. Find the equation of the circle with..

A. center \((-2, 3)\) and diameter = 8

\((x - h)^2 + (y - k)^2 = r^2\)

\((x - -2)^2 + (y - 3)^2 = \left(\frac{8}{2}\right)^2 \rightarrow (x + 2)^2 + (y - 3)^2 = 16\)

B. center \((6, -5)\) and passes through the point \((1, 7)\)

\((x - h)^2 + (y - k)^2 = r^2\)

\((x - 6)^2 + (y - -5)^2 = r^2 \rightarrow (x - 6)^2 + (y - 5)^2 = r^2\) since we are told that the point \((1, 7)\) is on the circle the substituting those coordinates into the equation of the circle must make it true.

\((x - 6)^2 + (y - 5)^2 = r^2 \rightarrow (1 - 6)^2 + (7 - 5)^2 = r^2 \rightarrow (-5)^2 + (12)^2 = r^2 \rightarrow 25 + 144 = r^2 \rightarrow r^2 = 169\)

\([x - 6]^2 + (y + 5)^2 = 169\)
C. center $(-3,2)$ and tangent (touching at just one point) to the x-axis.

\[(x-h)^2 + (y-k)^2 = r^2\]  \hspace{1cm} \text{In order for the circle to be tangent to the x-axis it must touch immediately below the given center (draw a picture to see why). This means the radius MUST be 2.}

\[(x-3)^2 + (y-2)^2 = (2)^2 \rightarrow (x+3)^2 + (y-2)^2 = 4\]

D. the points $(7,13)$ and $(-3,-11)$ are the endpoints of a diameter.

To find the center just use the midpoint formula on the endpoints of the diameter then to find the radius choose one of the two given points to substitute into your equation (like in part B above).

\[(x-h)^2 + (y-k)^2 = r^2\]  \hspace{1cm} (7,13)  \hspace{1cm} (-3,-11) \rightarrow (h,k) = \left(\frac{7+(-3)}{2}, \frac{13+(-11)}{2}\right) = (2,1)\]

\[(x-2)^2 + (y-1)^2 = r^2\]

\[(7-2)^2 + (13-1)^2 = r^2 \rightarrow (5)^2 + (12)^2 = r^2 \rightarrow 25+144 = r^2 \rightarrow r^2 = 169\]

\[\boxed{(x-2)^2 + (y-1)^2 = 169}\]

4. The midpoint formula states that the point half-way between $(x_1, y_1)$ \textit{and} $(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

A) Show that the distance from $(x_1, y_1)$ to $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the same as the distance from $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to $(x_2, y_2)$.

\[d\left[(x_1, y_1) \text{ to } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right] = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2} = \sqrt{\left(\frac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_1}{2}\right)^2}\]

\[= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}\]

\[d\left[\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \text{ to } (x_2, y_2)\right] = \sqrt{\left(\frac{x_1 + x_2}{2} - x_2\right)^2 + \left(\frac{y_1 + y_2}{2} - y_2\right)^2} = \sqrt{\left(\frac{x_1 + x_2 - 2x_2}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_2}{2}\right)^2}\]

\[= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}\]

See the two expressions for the distances are identical!
B) Is what you did in part A) sufficient to prove the midpoint formula? If no, explain what else would have to be done (and why) and prove it.

NO it is not! Consider the picture to the right. The distance from A to C equals the distance from C to B BUT C is NOT the midpoint of the segment AB.

What we will need to show from part A is that

\[ d\left( (x_1, y_1) \text{ to } \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \right) + d\left( \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \text{ to } (x_2, y_2) \right) = d\left( (x_1, y_1) \text{ to } (x_2, y_2) \right) \]

i.e. Dist AC + Dist CB = Dist AB then we will know that C is the midpoint of AB.

\[
\begin{align*}
\sqrt{\frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4}} + \sqrt{\frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4}} & = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\
\sqrt{\frac{(x_2-x_1)^2}{2} + (y_2-y_1)^2} + \sqrt{\frac{(x_2-x_1)^2}{2} + (y_2-y_1)^2} & = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}
\end{align*}
\]

So indeed it is and we have now proven the midpoint formula!