1. State whether the following statements are TRUE or FALSE.

A) If \( \mathbf{u} \) and \( \mathbf{v} \) are vectors such that \( \mathbf{u} \cdot \mathbf{v} = 0 \), then the vectors are perpendicular. \( \text{TRUE} \)

B) If \( \mathbf{u} \) and \( \mathbf{v} \) are vectors such that \( \mathbf{u} \times \mathbf{v} = 0 \), then the vectors are parallel. \( \text{FALSE} \)

C) The line \( L_1 \) given by the symmetric equations \( \frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{3} \) is parallel to the vector \( \mathbf{v} = <1, -1, 2> \). \( \text{FALSE} \)

D) If two planes are perpendicular to a third plane, then the two planes are parallel. \( \text{FALSE} \)

2. Given the vectors \( \mathbf{u} = <1, 2, 3> \) and \( \mathbf{v} = <-2, -1, 2> \), and \( \mathbf{w} = <3, 4, 5> \) find the following:

A) \( \mathbf{u} - 2\mathbf{v} \)
\[ <1, 2, 3> - 2 <2, -1, 2> = <1 - 4, 2 + 2, 3 - 4> = <-3, 4, -1> \]

B) \( \mathbf{u} \cdot \mathbf{v} \)
\[ (1)(-2) + (2)(-1) + (3)(2) = 1 - 2 + 6 = 5 \]

C) \( \mathbf{u} \times \mathbf{v} \)
\[ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 2 & -1 & 2 \end{vmatrix} = (4 - 3)\mathbf{i} - (2 - 6)\mathbf{j} + (1 - 4)\mathbf{k} = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \]

D) \( \text{PROJ}_\mathbf{v} \mathbf{u} \)
\[ \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{v}||^2} \mathbf{v} = \frac{5}{9} <2, -1, 2> \]

E) The area of the parallelogram formed by \( \mathbf{u} \) and \( \mathbf{v} \).
\[ ||\mathbf{u} \times \mathbf{v}|| = \sqrt{49 + 16 + 25} = \sqrt{80} \approx 3.39 \]

F) A 45 pound force acts in the direction of \( \mathbf{v} \), find the vector component form of this force.
\[ 45 \cdot \frac{\mathbf{v}}{||\mathbf{v}||} = \frac{45}{3} <2, -1, 2> = 15 <2, -1, 2> = <30, -15, 30> \]

G) EXTRA CREDIT –
Find the volume of the parallelepiped formed by the vectors \( \mathbf{u}, \mathbf{v}, \) and \( \mathbf{w} \).
\[ (\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})) = |<3, 4, 5> \cdot <7, 9, -5>| = |21 + 36 + 25| = 82 \]
3. Two forces of 60 pounds and 90 pounds act upon an object with angles from the horizontal of 135° and 30° respectively. Find the resulting force. To receive full credit, your final answer should not be in terms of the angles, use exact values.

\[
\begin{align*}
F_1 &= 60 \left< \cos(135°), \sin(135°) \right> \\
F_2 &= 90 \left< \cos(20°), \sin(20°) \right> \\
F_1 &= 60 \left< -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right> \\
F_2 &= 90 \left< \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right> \\
F_2 &= <45\sqrt{2}, 45> \\
F &= F_1 + F_2 = <30\sqrt{2} + 45\sqrt{2}, 30\sqrt{2} + 45> \\
\end{align*}
\]

4. Find the work done in moving an object from the point (3,-1,0) to the point (2,3,1) if the force is given by \( \mathbf{F} = <5, 6, -2> \).

\[
\text{work} = \mathbf{F} \cdot \mathbf{D} = <5, 6, -2> \cdot <-1, 4, 1> = 5 + 24 + (-2) = 27
\]

5. **SET UP ONLY** the vector component form of the force vector and the distance vector you would use to calculate the torque in the diagram below about the point A. Let \( \theta = 45° \) and to receive full credit, your final answer should not be in terms of the angles, use exact values. **YOU DO NOT NEED TO FIND THE TORQUE.** Just set up the vectors.

\[
\begin{align*}
F_1 &= 180 \left< \cos(225°), \sin(225°) \right> \\
F_2 &= 180 \left< -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right> \\
F_3 &= <90\sqrt{2}, -90\sqrt{2}> \\
\end{align*}
\]

6. Find the parametric equations of the line that passes through the point \( \mathbf{P} \) and is parallel to the line given by the parametric equations: \( x = -2t + 1, y = 3t + 2, z = t - 2 \)

\[
\begin{align*}
x &= -2t + 1 \\
y &= 3t + 2 \\
z &= t + 3
\end{align*}
\]

7. Find the parametric equations of the line that contains the points \( \mathbf{P} \) and that is perpendicular to the plane given by \( 2x - y + 3z = 6 \)

\[
\begin{align*}
\mathbf{N} &= <2, -1, 3> \\
\mathbf{D} &= <2t + 1, -t + 2, 3t + 3>
\end{align*}
\]
8. Find the parametric equations of the line that contains the point \((-1, 4, -2)\) and is parallel to the xy-plane and xz plane.

\[
\begin{align*}
\vec{v} &= \langle 2-1, 1-1, 3-1 \rangle \\
\vec{w} &= \langle 1, 0, -2 \rangle \\
\vec{u} &= \langle 3-1, 2-1, 1-1 \rangle \\
\vec{r} &= \langle 2, 1, 0 \rangle \\
\vec{n} &= \vec{v} \times \vec{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ 1 & 0 & -2 \end{vmatrix} \\
\vec{n} &= \langle 2, 8, -2 \rangle \\
\end{align*}
\]

\[
\begin{align*}
x &= t - 1 \\
y &= 4 \\
z &= -2 \\
\end{align*}
\]

9. Find an equation of the plane that contains the points \((1, 1, 1)\), \((2, 1, -3)\), and \((3, 2, 1)\).

\[
\begin{align*}
\vec{a} &= \langle 2-1, 1-1, 3-1 \rangle \\
\vec{b} &= \langle 1, 0, -2 \rangle \\
\vec{c} &= \langle 3-1, 2-1, 1-1 \rangle \\
\vec{r} &= \langle 2, 1, 0 \rangle \\
\vec{n} &= \langle 5, -8, 1 \rangle \\
\end{align*}
\]

\[
\begin{align*}
x - 8y + z &= k. \\
5x - 8y + z &= -2. \\
\end{align*}
\]

10. Find an equation of the plane that contains the point \((1, 2, 3)\), and is parallel to the plane given by \(2x - 5y + 3z = 30\).

\[
\begin{align*}
\vec{n} &= \langle 2, -5, 3 \rangle \\
\end{align*}
\]

\[
\begin{align*}
x - 5y + 3z &= k \\
(1, 2, 3) \\
2(-1) - 5(2) + 3(3) &= k \\
2 - 10 + 9 &= k \\
k &= 1 \\
\end{align*}
\]

11. Find an equation of the plane that contains the point \((-1, 4, -2)\), and is parallel to the yz-plane.

\[
\vec{n} = \langle 1, 0, 0 \rangle
\]

\[
\begin{align*}
A &+ B + Cz = k. \\
1x + 0y + 0z &= k \\
1x &= k \\
(1, -1) &= k \\
k &= -1 \\
\end{align*}
\]

12. Find the point of intersection between the line \(x = 2t - 1, y = 3t, z = -t + 2\) and the line \(x = s - 1, y = s + 1, z = 2s - 3\).

\[
\begin{align*}
&\quad \mathbf{t} = 1 \quad \mathbf{t} = 1 \\
&\quad \mathbf{x} = 2(1) - 1 = 1 \\
&\quad \mathbf{y} = 3(1) = 3 \\
&\quad \mathbf{z} = -1 + 2 = 1 \\
&\quad \mathbf{s} = 2 \\
&\end{align*}
\]

\[
\begin{align*}
x &= 2t - 1 \\
y &= 2 + 1 = 3 \\
z &= 2(2) - 3 - 1 = 1 \\
\end{align*}
\]

Point of int. \((1, 3, 1)\)
13. Find the parametric equations for the line of intersection of the planes \(2x - y + 3z = 6\) and \(x + y - 2z = -2\).

\[\begin{align*}
\text{Direction vector:} & \quad \vec{v} = \langle -1, 1, 3 \rangle \\
\begin{vmatrix}
2 & -1 & 3 \\
1 & 1 & -2 \\
1 & 1 & 3
\end{vmatrix} & = (2+3)\hat{i} - (-4-3)\hat{j} + (2-1)\hat{k} \\
& = \hat{i} + 7\hat{j} + 3\hat{k}
\end{align*}\]

Let \(x = 0\) then \(z = -3\).

\[\begin{align*}
P & = (0, 3, -3) \\
2(0) - y + 3(0) & = 6 \\
-2y & = 6 \\
y & = -3
\end{align*}\]

14. Find the distance between the point \((3, 2, 1)\) and the line given by the parametric equations \(x = t, y = t + 3, z = t + 4\).

\[\begin{align*}
P & = (0, 3, 4) \\
Q & = (3, 2, 1) \\
\vec{PQ} & = \langle 3, -1, -3 \rangle \\
\vec{n} & = \langle 1, -1, 1 \rangle \\
\|\vec{PQ} \times \vec{n}\| & = \sqrt{14 + 36 + 16} \\
& = \sqrt{76}
\end{align*}\]

\[D = \frac{\sqrt{76}}{3}\]

15. Find the distance between the point \((3,2,1)\) and the plane given by \(2x - y + z = 4\).

\[\begin{align*}
P & = (0, 0, 4) \\
Q & = (3, 2, 1) \\
\vec{PQ} & = \langle 3, 2, -3 \rangle \\
\vec{n} & = \langle 2, -1, 1 \rangle
\end{align*}\]

\[D = \frac{1}{\sqrt{6}}\]

16. Find the distance between the plane given by the equation \(x - 3y + 4z = 12\) and the line given by the equations \(x = 7t + 5, y = t + 1, z = -t + 2\). (They do not intersect, do not worry about showing that)

\[\begin{align*}
P & = (0, 0, 3) \\
Q & = (5, 1, 2) \\
\vec{PQ} & = \langle 5, 1, -1 \rangle \\
\vec{n} & = \langle -1, -3, 4 \rangle
\end{align*}\]

\[D = \frac{2}{\sqrt{26}}\]