Section 2.3 Volume

Volume is the amount of 3-dimensional space an object occupies.

Consider the diagram below.

For this example the volume is \(4\text{cm} \times 3\text{cm} \times 5\text{cm} = 60\text{ cm}^3\)

This figure is composed of 60 cubes that are each 1 cm by 1 cm by 1 cm. They each have a volume of 1 cubic centimeter
Here are the volume formulas we will need for this section.

**Volume Formulas**

- **Cube**
  
  \[ V = s^3 \]

- **Rectangular Prism**
  
  \[ V = lwh \text{ or } V = Bh \]

- **Sphere**
  
  \[ V = \frac{4}{3} \pi r^3 \]

- **Right Circular Cylinder**
  
  \[ V = \pi^2 h \]

- **Right Circular Cone**
  
  \[ V = \frac{1}{3} \pi r^2 h \]

- **Right Square Pyramid**
  
  \[ V = \frac{1}{3} s^2 h \]
Example: Find the volume.

Step 1: Identify the name of the shape and the formula to use.

This is a rectangular pyramid.

Formula: \( V = \frac{1}{3}Bh \text{ where } B = lw \)

We can rewrite the volume formula as follows:

\[ V = \frac{1}{3}lwh \]

Step 2: Assign values to each variable and plug the numbers in the formula.

\( l = 8 \text{ in}, \ w = 7 \text{ in}, \ h = 6 \text{ in} \) (the values of “l” and “w” are interchangeable, although we usually call the longer side in a rectangle the length.)

\[ V = \frac{1}{3}(8\text{in})(7\text{in})(6\text{in}) \]

Answer: Volume = 112 in\(^3\) (my units are cubic inches)
Example: Find the volume.

Step 1: Identify the name of the shape and the formula to use.
This is a triangular prism.

Formula: \( V = \frac{1}{2} bhl \)

Step 2: Assign values to each of the variables.
\( b = 15 \text{ cm}, \ h = 8 \text{ cm}, \ l = 12 \text{ cm} \)

\[ V = \frac{1}{2} (15 \text{ cm}) (8 \text{ cm}) (12 \text{ cm}) \]

Answer: Volume 720 cm\(^3\)

Example: Find the volume.

Step 1: Identify the name of the shape and the formula to use.
This is a right circular cone.

Formula: \( V = \frac{1}{3} \pi r^2 h \)

Step 2: Assign values to each variable and plug the numbers in the formula.
\( r = 5.7 \text{ cm}, \ h = 12 \text{ cm} \)

\[ V = \frac{1}{3} \pi (5.7 \text{ cm})^2 (12 \text{ cm}) \]

Answer: Volume = 22.8\(\pi\text{cm}^3\)
**Example:** Find the volume.

![Diagram of a combination of a cone and a sphere.](image)

Step 1: Identify the shape.

This is a combination of two shapes. The top is one half of a sphere. The bottom is a right circular cone.

Formulas:

Volume of right circular cone: \( V = \frac{1}{3} \pi r^2 h \)

Volume of a sphere \( V = \frac{4}{3} \pi r^3 \)

Volume of half of a sphere
\( V = \frac{1}{2} \times \frac{4}{3} \pi r^3 \) (I need to multiply the volume by \( \frac{1}{2} \)) (Cancel the 4 and 2)

Volume of half a sphere = \( V = \frac{2}{3} \pi r^3 \)

Step 2: Assign values and plug numbers in the formula.

Cone \( r = 4 \text{ cm}, h = 10 \text{ cm} \)

Sphere \( r = 4 \text{ cm} \)

Cone part + Sphere part
\[ V = \frac{1}{3} \pi (4\text{cm})^2 (10\text{cm}) + \left(\frac{2}{3} \pi (4\text{cm})^3 \right) \]

\[ V = \frac{1}{3} \pi (160\text{cm}^3) + \left(\frac{2}{3} \pi 64\text{cm}^3 \right) \]

\[ V = \frac{160}{3} \pi \text{cm}^3 + \frac{128}{3} \pi \text{cm}^3 \]

\[ V = \frac{288}{3} \pi \text{cm}^3 \]

Answer: Volume = \( 96 \pi \text{cm}^3 \)
Homework #1-30: Find the volume. Leave your answer in terms of $\pi$ when appropriate.

1) [Diagram of a rectangular prism with dimensions 4.0 cm, 6.4 cm, and 9.0 cm.]

2) [Diagram of a rectangular prism with dimensions 3 cm, 10 cm, and 2 cm.]

3) [Diagram of a rectangular prism with dimensions 2 feet, 6 feet, and 3 feet.]

4) Assume measurements are in feet.

5) [Diagram of a cylinder with dimensions 12 cm and 4 cm.]

6) [Diagram of a cylinder with dimensions 2 cm and 4 cm.]
11) Assume measurements are in centimeters.
13) Assume the base of the pyramid is a square. Leave your answer in fraction form.

14) Assume that the units are given in meters. Round your answer to 2 decimals, if appropriate.

15) This is a square pyramid. The base is a 4 cm by 4 cm square.

16) 

17) 

18) 

19) Write your answer as a fraction in terms of $\pi$.

20) Write your answer as a fraction in terms of $\pi$. 
21) Write your answer as a fraction in terms of $\pi$.

22) Write your answer as a fraction in terms of $\pi$.

23) Hint you will need to subtract to find the height of the cone. Write your answer as a fraction in terms of $\pi$.

24) Write your answer as a fraction in terms of $\pi$.

25) Write your answer in terms of $\pi$.

26) Write your answer as a fraction in terms of $\pi$. 
27) Assume lengths are given in feet.

28) Assume lengths are given in inches.

29) Assume lengths are given in inches.

30) Assume lengths are given in inches.

31) Assume lengths are given in meters.

32) Assume lengths are given in millimeters.
Answers: 1) 194.4 cm$^3$ 3) 36 ft$^3$ 5) 192$\pi$ cm$^3$ 7) 4.608$\pi$ ft$^3$ 9) 96$\pi$ cm$^3$ 11) 960$\pi$ cm$^3$
13) $\frac{175}{3}$ m$^3$ 15) 48 cm$^3$ 17) 112 in$^3$ 19) $\frac{10976}{3}$ m$^3$ 21) $\frac{32}{3}$ in$^3$ 23) 60840$\pi$ ft$^3$
25) $\frac{475}{3}$ in$^3$ 27) 24000 ft$^3$ 29) 1428 in$^3$ 31) 24000 mm$^3$