Section 3.3: Measures of Central Tendency

An “average” is a number that is representative of a group of data. There are at least four different averages: MEAN, the MEDIAN, the MODE, and the MIDRANGE.

The **Mean** (which is given the symbol $\bar{x}$), is the sum of the data divided by the number of pieces of data. $\bar{x} = \frac{\sum x}{n}$

**Example:** Five people are waiting at a Jack in the Box drive through have ages 27, 18, 48, 34 and 48. What is the mean age of this group?

I just add up the 5 ages, and divide by 5.

$$\text{mean} = \frac{27 + 18 + 48 + 34 + 48}{5} = 35$$

Answer: The mean age is 35.

The **Median** is the value in the middle of an ordered set of data.

**Example:** Find the median age of the customers waiting in line at the Jack in the Box drive through.

I need to put the numbers in order. I will place them from smallest to largest. It is okay to place them from largest to smallest.

Ranked: 18 27 34 48 48

once ranked, remove the first and the last number to get:

27 34 48

Repeat and remove the first and the last number and you are left with the median.

Median = the “middle” value = 34

Answer: Median = 34
If there is an even number of data points you need to average to find the median value.

**Example:** Find the median for this set of data.

2, 3, 3, 5, 7, 7, 9, 12

First remove the first and the last number to get:

3, 3, 5, 7, 7, 9

Then repeat and remove the first and the last number to get:

3, 5, 7, 7

Repeat again to get:

5, 7

There are two numbers left. When this happens, the median is the average of these numbers.

Answer: Median = \( \frac{5 + 7}{2} = 6 \)

The **Mode** is the piece of data that occurs most often.

**Example:** Find the mode of the ages of the people in the Jack in the Box drive through. Here are the ages again: 27, 18, 48, 34 and 48

Mode = 48 (it occurs most often in the list)

If there is more than one value that occurs “most often” then we list both of them as Modes.

**Example:** Find the Mode of 2, 2, 3, 3, 3, 5, 7, 7, 9, 12

3 and 7 both occur 3 times. They are both modes. 2 is not a mode as it doesn’t occur the maximum number of times.

Answer: Mode = 3, 7 (We say this data is bi-modal.)
Example: Find the mode: 1, 3, 5, 7, 9

Answer: This has no mode. (Each number occurs the same number of times.)

The Midrange = $\frac{\text{lowest number} + \text{highest number}}{2}$

Example: Find the midrange of Jack in the box drive though data. Here are the ages again: 27, 18, 48, 34 and 48

Answer: Midrange = $\frac{18 + 48}{2} = 33$

Here is another type of problem you will be asked to solve in the homework in this section.

Example: Cindy’s mean average on 7 exams is 80. Find the sum of her scores.

Just multiply the number of tests times the average score.

Sum = 7 * 80 = 560

Answer: Sum of her test scores 560 (This assumes each test is worth 100 points.)

Example: A mean average of 90 or greater on five exams is needed for a final grade of A in a certain course. Thurston’s first four exam grades were 93, 85, 95 and 90.

a) What grade does Thurston need on the fifth exam to get an A for the course?

I need to find the total points needed for an A and his total points so far.

Total points needed for an A: 90 * 5 = 450
Total points so far: 93 + 85 + 95 + 90 = 363

Now subtract the totals to find an answer to the question: 450 – 363 = 87

Answer: 87 points

b) An average of 80 is needed to get a B? What grade does Thurston need to get a B?

I need to find the total points needed for a B and his total points so far.

Total points needed for a B: 80 * 5 = 400
Total points so far: 93 + 85 + 95 + 90 = 363
Subtract these amounts to arrive at an answer: 400 – 363 = 37

Answer: 37 points
c) If his lowest grade of the exams already taken is to be dropped, what grade must he receive on his last exam to get an A?

I need to find the total points needed for an A and his total points so far.

Total points needed for an A: $90 \times 4 = 360$ (only keep 4 best test scores)
Total points so far: $93 + 95 + 90 = 278$ (I dropped the 85.)

Subtract the amounts to get an answer: $360 - 278 = 82$

Answer: 82 points

Homework #1-8: Determine the mean, median, mode, and midrange for the given data.

1) 2, 3, 5, 5, 20
2) 1, 2, 3, 5, 6, 6, 7
3) 11, 13, 15, 12, 18, 5, 1, 13
4) 21, 25, 29, 27, 15, 21, 28, 13
5) 1, 1, 1, 5, 5, 5
6) 2, 2, 4, 5, 5, 6
7) 1, 2, 3, 4, 9, 8, 7 (round mean to 2 decimals)
8) 6, 7, 8, 9, 5, 4, 3

9) Greg’s mean average on five exams is 81. Find the sum of his scores.

10) Peter’s mean average on 6 exams is 74. Find the sum of his scores.

11) Jan’s mean average on four exams is 90. Find the sum of her scores.

12) Bobby’s mean average on 3 exams is 92. Find the sum of his scores.

13) Lemont plays high school basketball. After 20 games Lemont averaged 18.5 points per game. How many total points did Lemont score?

14) Grady’s car gets 25 mpg. On a recent trip Grady used 10 gallons of gas. How far did he travel?

15) A new sub shop sells 70 sandwiches each day on average. How many sandwiches do they sell in a week?

16) A new car salesman sells 5 cars on average each week. How many cars will he sell in a year, assume he takes two weeks of vacation?
17) A mean average of 80 or greater for five exams is needed for a final grade of B in a course. Thurston’s first four exam grades were 73, 69, 85 and 80.

a) What grade does Thurston need on the fifth exam to get a B for the course?

b) An average of 70 is needed to pass the course. What grade does Thurston need to pass the course?

c) If his lowest grade of the exams already taken is to be dropped, what grade must he receive on his last exam to pass the class?

18) A mean average of 70 on 6 exams is needed to pass a course. On her first 5 exams, Ellie received grades of 55, 77, 72, 64, and 95.

a) What grade must she receive on her last exam to pass the course?

b) An average of 80 is needed to get a B in the course. Is it possible for Ellie to get a B? If so, what grade must she receive on the 6th exam?

c) If her lowest grade of the exams already taken is to be dropped, what grade must she receive on her last exam to pass the class?

19) A mean average of 80 or greater for five exams is needed for a final grade of B in a course. Thurston’s first four exam grades were 73, 69, 85 and 80. The first four tests were worth 100 points, the fifth exam is worth 200 points.

a) What grade does Thurston need on the fifth exam to get a B for the course?

b) An average of 70 is needed to pass the course. What grade does Thurston need to pass the course?

20) A mean average of 70 on 6 exams is needed to pass a course. On her first 5 exams, Ellie received grades of 55, 77, 72, 64, and 95. The first 5 exams were worth 100 points and the last exam is worth 200 points.

a) What grade must she receive on her last exam to pass the course?

b) An average of 80 is needed to get a B in the course. Is it possible for Ellie to get a B? If so, what grade must she receive on the 6th exam?
Answers: 1) mean 7, median 5, mode 5, midrange 11
3) mean 11, median 12.5, mode 13, midrange 9.5
5) mean 3, median 3, mode 1.5, midrange 3
7) mean 4.86, median 4, mode none, midrange 5
9) 405 11) 360 13) 370 15) 490 17a) 93 17b) 43 17c) 42
19a) 173 19b) 113