Chapter 3 Review

#1-3: Determine whether the equation defines y as a function of x. Hint, solve the equation for y and sketch a graph using your calculator, then apply the vertical line test.

1)  \( y = (x-3)^2 + 4 \)

Graph passes the vertical line test.
Answer #1: \( y \) is a function of \( x \).

You may just say (yes, or the graph is a function).

2)  \( y^2 = x + 4 \)

First solve for \( y \)

\[ \sqrt{y^2} = \pm \sqrt{x + 4} \]

\[ y = \pm \sqrt{x + 4} \]

Graph \( y_1 = \sqrt{x + 4} \) and \( y_2 = -\sqrt{x + 4} \)

This graph fails the vertical line test, so it is not a function.

Answer #2: \( Y \) is not a function of \( x \), or just say not a function
3) \( y = \sqrt{x - 1} \)

Answer #1: \( y \) is a function of \( x \).
You may just say (yes, or the graph is a function).

#4 – 6: Determine the domain and range of each function, write your answer in interval notation when appropriate.

4) Domain will be the \( x \)-coordinates of each point, the Range will be the \( y \)-coordinates.

Answer
Domain = \{3, 4, 5, 7\}
Range = \{-1, 2, 6\}

5) Domain [\( x \)-coordinate far left point, \( x \)-coord far right point]
Range [\( y \)-coordinate of bottom point, \( y \)-coordinate of top point]
Answer
Domain [0, 5]
Range [-5, 4]
6) Graph needs to be extended to the left. The graph goes to the far left edge of the x-axis which has an x-coordinate of $-\infty$.

The domain starts at the far left edge of the x-axis or $x = -\infty$.

The domain ends at the point (0,2) end at $x = 0$.

Domain $(-\infty, 0]$.

Graph extends to the bottom of the y-axis. The range starts at the bottom of the graph which is at $y = -\infty$.

The range ends at the point (0,2) or more specifically at $y = 0$.

Range $(-\infty, 2]$. 
#7-9: Use algebra to find the domain of each function. Write your answer in interval notation

7) \( f(x) = \frac{x-4}{x^2+6x-7} \)

Ignore the numerator

Solve denominator = 0

\( x^2 + 6x - 7 = 0 \)

\( (x + 7)(x - 1) = 0 \)

\( x + 7 = 0 \) \hspace{1cm} \( x - 1 = 0 \)

\( x = -7 \) \hspace{1cm} \( x = 1 \)

Plot \( x = -\infty, -7, 1, \infty \) on a number line and create intervals with all round parenthesis.

\((-\infty, -7) \quad (-7, 1) \quad (1, \infty)\)

\( \infty \quad -7 \quad 1 \quad -\infty \)

Answer #7: domain \((-\infty, -7) \cup (-7, 1) \cup (1, \infty)\)

8) \( f(x) = \sqrt{x + 5} \)

\( x + 5 \geq 0 \)

\(-5 \quad -5\)

\( x \geq -5 \) This is the answer, but it is not in interval notation.

Answer #8: \([-5, \infty)\)

9) \( f(x) = 2x - 6 \)

This is a polynomial and there is no algebra required to find the domain.

Answer #9: domain \((-\infty, \infty)\)
#10–13: let \( f(x) = x^2 + 3x - 4 \) and \( g(x) = 7x - 7 \), find the following

10) \( (f - g)(x) = f(x) - g(x) \)
\[
(f - g)(x) = (x^2 + 3x - 4) - (7x - 7)
\]
\[
(f - g)(x) = x^2 + 3x - 4 - 7x + 7
\]
\[
(f - g)(x) = x^2 - 4x + 3
\]
Answer #10: \( (f - g)(x) = x^2 - 4x + 3 \) it would be better to write \( (f - g)(x) = (x-1)(x-3) \)

11) \( (f \circ g)(x) = f(g(x)) \)
\[
(f \circ g)(x) = (g(x))^2 + 3(g(x)) - 4
\]
\[
(f \circ g)(x) = (7x - 7)^2 + 3(7x-7) - 4
\]
\[
(f \circ g)(x) = (7x - 7)(7x - 7) + 3(7x - 7) - 4
\]
\[
(f \circ g)(x) = 49x^2 - 49x - 49x + 49 + 21x - 21 - 4
\]
Answer #11: \( (f \circ g)(x) = 49x^2 - 77x + 24 \)

12) \( (f + g)(5) = f(5) + g(5) \)
\[
(f + g)(5) = (5^2 + 3(5) - 4) + (7(5) - 7)
\]
\[
(f + g)(5) = 36 + 28
\]
Answer: \( (f + g)(5) = 64 \)

13) \( (g \circ f)(4) \)
First find \( (g \circ f)(x) = 7(f(x)) - 7 = 7(x^2 + 3x - 4) - 7 = 7x^2 + 21x - 28 - 7 \)
\[
(g \circ f)(x) = 7x^2 + 21x - 35
\]
Next plug in 4 for x.
\[
(g \circ f)(4) = 7(4)^2 + 21(4) - 35 = 161
\]
Answer #13: \( (g \circ f)(4) = 161 \)
14) Find the difference quotient; that is find \( \frac{f(x + h) - f(x)}{h} \) where \( f(x) = x^2 + 5x + 2 \)

Find \( f(x + h) = (x + h)^2 + 5(x + h) + 2 \)

\f(x + h) = (x +h)(x+h) + 5(x + h) + 2

\( f(x + h) = x^2 + 1xh + 1xh + h^2 + 5x + 5h + 2 \)

\( f(x + h) = x^2 + 2xh + h^2 + 5x + 5h + 2 \)

Now replace the symbols in the numerator of the difference quotient.

\[ \frac{f(x + h) - f(x)}{h} = \frac{(x^2 + 2xh + h^2 + 5x + 5h + 2) - (x^2 + 5x + 2)}{h} \]

\[ \frac{f(x + h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 + 5x + 5h + 2 - x^2 - 5x - 2}{h} \]

\[ \frac{f(x + h) - f(x)}{h} = \frac{2xh + h^2 + 5h}{h} \]

\[ \frac{f(x + h) - f(x)}{h} = \frac{h(2x + h + 5)}{h} \]

Answer #14: \( \frac{f(x + h) - f(x)}{h} = 2x + h + 5 \)
15) find the x-intercepts
asking for points on the x-axis.
Answer (-4, 0) and (0, 0)

16) find the y-intercept
Asking for the point on the y-axis
Answer (0,0)

17) for what values of x is h(x) = -4
Wants the x-value of any point that has a y = -4
Answer x = -2

18) find h(-4)
Wants the x-coordinate of any point that has -4 for and x-coordinate.
Answer h(-4) = 0 you may also write y = 0

19) what is the domain of h
Domain = [x-coord far left point, x=coord far right point)
Graph has to be extended up and to the right, so the end of the domain is \( \infty \)
answer domain = [-4, \( \infty \)]
20) what is the range of h

Range = [y-coord bottom point, y=coord top point)
Graph has to be extended up and to the right, so the end of the range is \( \infty \)
answer range = [-4, \( \infty \))

21) Find the following:
a) the interval(s) where the function graphed is increasing

The graph increases from the beginning which is \( x = -\infty \)
To the first turning point \( x = -3 \)
It also increases from the second turning point
\( x = -1 \) to \( x = \infty \)
Answer 21a) increasing \((-\infty, -3) \cup (-1, \infty)\)

b) the interval(s) where the function graphed is decreasing

The graph is decreasing between the turning points \( x = -3 \) to \( x = -1 \)
Answer 21b) decreasing \((-3, -1)\)

c) The values of x (if any) where the function has a local maximum

The x-coord of the high point
21c) answer \( x = -3 \)

d) The local maximum value (if any)

Asking for y coord of max
21d) answer \( y = -8 \)

e) The values of x (if any) where the function has a local minimum

Asing for x-coord of low point.
Answer 21e) \( x = -1 \)

f) The local minimum values (if any)

Wants y-coord of low point.
Answer 21f) \( y = -12 \)
22) Find the following:
   a) the interval(s) where the function graphed is increasing
      The graph increases from the beginning which is \( x = -\infty \)
      To the vertex \( x = 3 \)
      Answer 22a) increasing \( (-\infty, 3) \)
   b) the interval(s) where the function graphed is decreasing
      The graph is decreasing from the vertex \( x = 3 \) to the end of the graph \( x = \infty \)
      Answer 22b) decreasing \( (3, \infty) \)
   c) The values of x (if any) where the function has a local maximum
      The x-coord of the vertex
      22c) answer \( x = 3 \)
   d) The local maximum value (if any)
      Asking for y coord of max
      22d) answer \( y = 4 \)
   e) The values of x (if any) where the function has a local minimum
      no low points – no mins
      Answer 22e) none
   f) The local minimum values (if any)
      Wants y-coord of low point.
      Answer 21f) Answer none
23) Find the average rate of change of \( f(x) = (3x - 2)^2 - 4 \) from 4 to 5

Create two points. We are given the x-coordinates and need to find y-coordinate.

\[
f(4) = (3(4) - 2)^2 - 4 \\
f(4) = (10)^2 - 4 \\
f(4) = 96
\]

First point \((4, 96)\)

\[
f(5) = (3(5) - 2)^2 - 4 \\
f(5) = (13)^2 - 4 \\
f(5) = 165
\]

Second point \((5, 165)\)

Average rate of change is the slope of the line that connects the points.

\[
\text{Average rate of change} = \frac{165 - 96}{5 - 4} = \frac{69}{1}
\]

Answer #23: Average rate of change = 69

24) Problem 24 has been deleted.
25) Let $f(x) = x^2$

25a) Sketch a graph of $f(x)$

25b) Find $4f(x) = 4x^2$

25c) Make a table of values and sketch a graph of $4f(x)$ on the same graph as the graph of $f(x)$

25d) Describe the transformation to obtain the graph of $4f(x)$ from the graph of $f(x)$

Graph is stretched, you may also say graph is narrower.
#26 – 30, write the function whose graph is the graph of \( f(x) = x^2 \), but is

26) Shifted to the left 4 units

Need a \((x+4)\) in a parenthesis

Answer #26: \( f(x+4) = (x+4)^2 \)

27) Shifted to the right 4 units

Need a \((x+4)\) in a parenthesis

Answer #27: \( f(x-4) = (x-4)^2 \)

28) Shifted to the right 5 units and up 3 units

Need and \((x - 5)\) in a parenthesis to shif right 5

Need an + 3 to move up after the parenthesis

Answer #28: \( f(x - 5) + 3 = (x - 5)^2 + 3 \)

28) Shifted to the left 2 units and down 3 units

Need a \((x + 2)\) to move left

Need a minus 3 after parenthesis to move down.

Answer #28: \( f(x + 2) - 3 = (x + 2)^2 - 3 \)

29) Reflected over the x-axis

\(-f(x)\) will reflect over x-axis

Answer #29: \(-f(x) = -x^2\)

30) Reflected over the y-axis

\(f(-x)\) will reflect over y-axis

\(f(-x) = (-x)^2\)
31) A campground owner has 2000 feet of fencing. He wants to enclose a rectangular field bordering a river. Let \( W \) represent the width of the field. Follow these steps to find the dimensions of the field that yields the largest area. (round all answers to 2 decimal places if needed)

31a) Write an equation for the length of the field

See video for diagram

\[ L + 2W = 2000 \]

Answer 31a: \( L = -2W + 2000 \)

31b) Write an equation for the area of the field.

\[ A = LW \]

\[ A = (-2W + 2000)W \]

Answer 31b: \( A = -2W^2 + 2000W \)

31c) Find the value of \( W \) leading to the maximum area

use \( \frac{-b}{2a} \) formula

width = \( \frac{-2000}{2(-2)} = 500 \)

Answer 31c: Width = 500 ft

31d) Find the value of \( L \) leading to the maximum area

\[ L = -2W + 2000 \]

\[ L = -2(500) + 2000 \]

\[ L = 1000 \]

Answer 31d: Length = 1000 ft

31e) Find the maximum area

\[ A = LW \]

\[ A = (500ft)(1000ft) \]

Answer 31e: \( A = 500,000 \text{ ft}^2 \)