1) A campground owner has 800 meters of fencing. He wants to enclose a rectangular field. Let \( w \) represent the width of the field. Follow these steps to find the dimensions of the field that yields the largest area.

Sketch a diagram.

\[ \text{width} = w \]
\[ \text{length} = l \]

a) Write an expression for the length of the field.
   The perimeter of the region to be fenced is given by
   \[ P = 2l + 2w \]
   \[ 800 = 2l + 2w \]
   Solve this for \( l \) to answer question.
   \[ 800 = 2l + 2w \]
   \[ -2w = -2w \]
   \[ \frac{800 - 2w}{2} = \frac{2l}{2} \]

**ANSWER:** \( 400 - w = l \)

b) Write an equation for the area of the field.
   Area of region = \( A = lw \)
   Replace \( l \) with \( (400 - w) \)

**ANSWER:** \( A = (400 - w)w \)

c) Find the value of \( w \) leading to the maximum area.

Graph \( y = (400 - x)x \)

\[ (200, 400000) \]

Find maximum point

\( (200, 400000) \)

\( x \)-coordinate of vertex gives optimal width

**ANSWER:** 200 meters wide
d) find the value of \( L \) leading to the maximum area

\[
\text{Find optimal length}
\]

\[
L = 400 - W
\]

\[
= 400 - (200)
\]

\[
= 200
\]

**ANSWER:** 200 meters long

---

e) find the maximum area

\( y \)-coordinate of vertex gives maximum area

\[
(200, 40000)
\]

**ANSWER:** Area = 40000 m²

---

3) A campground owner has 1400 meters of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river, and let \( W \) represent the width of the field. Follow these steps to find the dimensions of the field that yields the most area.

\[
P = L + 2W
\]

a) write an expression for the length of the field

replace \( P \) with 1400

\[
1400 = L + 2W
\]

**Solve for \( L \)**

\[
1400 - 2W = L
\]

b) write an equation for the area of the field

\[
A = LW
\]

\[
A = (1400 - 2W)W
\]
c) Find the value of $W$ leading to the maximum area.

Sketch the graph $y = (1400 - 2x)(x)$

- $W = 350$
- $x_{min} = 0$
- $x_{max} = 700$
- $y_{min} = 0$
- $x_{max} = 250000$

Value of $W$ that leads to maximum area

$\text{Width} = 350$ meters

---

d) Find the value of $L$ leading to the maximum area.

$L = 1400 - 2W$

- Replace $W = 350$
- $L = 1400 - 2(350)$
- $L = 1400 - 700$
- $L = 700$

Value of $L$ that leads to maximum area

$\text{Length} = 700$ meters

e) Find maximum area.

The $y$-coordinate of vertex is maximum area $(350, 245000)$

Maximum area that can be enclosed = $245,000$ square miles
5) A fence must be built to enclose a rectangular area of 20,000 square feet.
Fencing material costs $2.50 per foot for the two sides facing north and south (call these sides the length, and $3.20 per foot for the other two sides (call these sides the width). Follow these steps to find the cost of the least expensive fence.

\[ L \text{ cost } 2.50 \text{ ft} \]
\[ w \text{ cost } 3.20 \text{ ft} \]
\[ L \text{ cost } 2.50 \text{ ft} \]
\[ w \text{ cost } 3.20 \text{ ft} \]

a) Write an equation for the length of the field

\[ A = LW \]

I create an area formula because I know the desired area

\[ \frac{20000}{w} = L \]

b) Write an equation for the cost of the field

\[ \text{Cost} = 2.50(\text{total lengths}) + 3.20(\text{total widths}) \]
\[ = 2.50(2L) + 3.20(2w) \]

\[ C = 5L + 6.40W \]

c) Find the value of \( W \) leading to the minimum cost

\[ C = 5L + 6.40W \]

replace \( L \) with \( \frac{20000}{w} \)

\[ C = 5(\frac{20000}{w}) + 6.40W \]

\[ (125, 1600) \]

Value of \( W \) that gives lowest cost

\[ W = 125 \text{ ft.} \]
d) find the value of \( L \) leading to the minimum cost
\[
L = \frac{20000}{W}
\]
\[
L = \frac{20000}{125}
\]
substitute optimal width (125) in for \( W \)
\[
L = 160
\]
Value of \( L \) that gives lowest cost

\[\text{length} = 160 \text{ ft.}\]

e) find minimum cost

lowest cost is \( y \)-coordinate of vertex
\[
\text{cost} = \$1,600
\]

7) A fence must be built in a large field to enclose a rectangular area of 25,000 square meters. One side of the area is bounded by an existing fence; no fence is needed there. Material for the fence costs \$3.00 per meter for the two ends, and \$1.50 per meter for the side opposite the existing fence. Find the cost of the least expensive fence.

\[
\text{existing fence}
\]

width \( W \) 3.00 per meter

length \( L \) 1.50 per meter

\[\text{width} W\]

\[\text{width} W\]

\[\text{length} L\]

\[\text{length} L\]

a) Write an equation for the length of the field
\[
A = LW
\]
\[
\frac{25,600}{W} = LW
\]
solve for \( L \)
\[
\frac{25,600}{W} = L
\]

b) Write an equation for the cost of the field
\[
\text{cost} = 1.50L + 3.00(2W)
\]
\[
= 1.50L + 6.00W
\]
\[
C = 1.50 \left( \frac{25600}{W} \right) + 6.00W
\]
c) find the value of \(W\) leading to the minimum cost

Sketch graph, find minimum

\[
\begin{align*}
\text{Value of } W & \text{ that leads to lowest cost} \\
& W = 80 \text{ meters}
\end{align*}
\]

\[
\begin{align*}
\text{x min } & 0 \\
\text{x max } & 140 \\
\text{y min } & 0 \\
\text{y max } & 3000
\end{align*}
\]

\[
(80, 960)
\]

---

d) find the value of \(L\) leading to the minimum cost

\[
L = \frac{25600}{W}
\]

\[
= \frac{25600}{80}
\]

\[L = 320 \text{ meters}
\]

replace with optimal width

---

e) find the minimum cost

y-coordinate of vertex

\( (80, 960) \)

\[960 = \text{lowest cost} \]

---

d) An open box with a square base is to be made from a square piece of cardboard 10 inches on a side by cutting out a square (x inches by x inches) from each corner and turning up the sides. (Round to 2 decimals if needed)

\[
\begin{align*}
\text{completed box:} \\
\text{length} & = 10 - 2x \\
\text{width} & = 10 - 2x \\
\text{height} & = x
\end{align*}
\]
b) Write an equation for the volume of the box

\[ V = Lwh \]
\[ = (10-2x)(10-2x)(x) \]
\[ = (10-2x)^2(x) \]
\[ V = x(10-2x)^2 \]

---

c) Graph the volume function using your graphing calculator and find the value of \( x \) that makes \( V \) the largest.

\[ \text{Min } x: 0 \]
\[ \text{Max } x: 5 \]
\[ \text{Min } y: 0 \]
\[ \text{Max } y: 100^+ \]

\[ (1.65, 74.07) \]

_x-coordinate_

Cutting out a 1.65 inch square gives maximum volume.

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11) An open box is to be made by cutting a square corner of a 20 inch by 20 inch piece of metal then folding up the sides. What size square should be cut from each corner to maximize volume? (round to 2 decimals if needed)

a)

\[ \text{Length } = 20 - 2x \]
\[ \text{Width } = 20 - 2x \]
\[ \text{Height } = x \]
\[ \text{Volume } = Lwh \]

---

b) Write an equation for the volume of the box

\[ V = Lwh \]
\[ = (20 - 2x)(20 - 2x)(x) \]
\[ V = x(20 - 2x)^2 \]

c) Graph the volume function using your graphing calculator and find the value of \( x \) that makes \( V \) the largest (round to 2 decimal places if needed)

\[ \text{Min } x: 0 \]
\[ \text{Max } x: 10 \]
\[ \text{Min } y: 0 \]
\[ \text{Max } y: 1000 \]

\[ (3.33, 592.59) \]

Cutting a 3.33 inch square will maximize volume.