Section 4.1: Linear functions and their properties

# 1 – 10: Find
   a) slope
   b) y-intercept
   c) x-intercept (if any)
   d) sketch a graph
   e) Determine the interval(s) where the graph is increasing, decreasing or constant.

1) \( f(x) = 3x - 6 \)
2) \( f(x) = 2x - 10 \)
3) \( g(x) = -2x - 8 \)
4) \( g(x) = -4x - 8 \)
5) \( f(x) = 7 \)
6) \( f(x) = -2 \)
7) \( g(x) = \frac{2}{3}x - 4 \)
8) \( g(x) = \frac{3}{4}x - 6 \)
9) \( f(x) = \frac{-x}{3} + 6 \)
10) \( f(x) = \frac{-x}{3} + 6 \)

11) Suppose \( f(x) = 3x - 6 \) and \( g(x) = -2x + 4 \)
   a) Solve \( f(x) = 0 \)
   b) Solve \( f(x) > 0 \)
   c) Solve \( f(x) = g(x) \)
   d) Solve \( f(x) < g(x) \)

12) Suppose \( f(x) = -3x - 2 \) and \( g(x) = -2x + 8 \)
   a) Solve \( f(x) = 0 \)
   b) Solve \( f(x) > 0 \)
   c) Solve \( f(x) = g(x) \)
   d) Solve \( f(x) < g(x) \)

13) Suppose \( f(x) = x - 3 \) and \( g(x) = 2x + 4 \)
   a) Solve \( f(x) = 0 \)
   b) Solve \( f(x) > 0 \)
   c) Solve \( f(x) = g(x) \)
   d) Solve \( f(x) < g(x) \)

14) Suppose \( f(x) = 3x - 6 \) and \( g(x) = 4x + 4 \)
   a) Solve \( f(x) = 0 \)
   b) Solve \( f(x) > 0 \)
   c) Solve \( f(x) = g(x) \)
   d) Solve \( f(x) < g(x) \)
Section 4.1: Linear functions and their properties

15) A jeweler’s salary was $30,000 in 2015 and $33,000 in 2016. The jeweler’s salary follows a linear growth pattern. (Create points in the form year, salary and use 15 for the year 2015 and 16 for the year 2016)
   a) Create a linear function to model the data
   b) Use the function to estimate the jeweler’s salary in 2020.

16) OSU had 28,000 students in 2014 and 29,500 students in 2016. Assume that the growth is linear. (Create points in the form year, enrollment and use 14 for the year 2014 and 16 for the year 2016)
   a) Create a linear function to model the data
   b) Use the function to estimate the enrollment in 2021.

17) A sub shop purchases a used oven for $1,000. After 5 years the oven will need to be replaced, and will have a value of $200. Assume the value of the oven goes down by the same amount each year. (Create two points, let the x-coordinates represent the age and the y-coordinates the value.)
   a) Write a linear function giving the value of the equipment for the 5 years it will be in use.
   b) Use the function to find the value of the equipment after 3 years.

18) A school district purchases a high volume copier for $10,000. After 10 years the copier will be worth $500 and it will have to be replaced. Assume the value of the equipment goes down by the same amount each year. (Create two points, let the x-coordinates represent the age and the y-coordinates the value.)
   a) Write a linear function giving the value of the equipment for the 10 years it will be in use.
   b) Find the value of the equipment after 3 years.

19) A real estate office manages an apartment complex with 100 units. When the rent is $600 per month, all 100 units will be occupied. However, when the rent is $700 per month only 90 units will be occupied. Assume the relationship between the monthly rent and the number of units occupied is linear. (Create two points let the x-coordinates represent the price and the y coordinates the number of units rented.)
   a) Write an equation for the number of units rented y in terms of the price p.
   b) Use the equation to predict the number of units occupied when the price is $675. (round to the nearest unit if needed)
   c) What price would cause 80 units to be occupied?
Section 4.1: Linear functions and their properties

20) A real estate office manages an apartment complex with 1000 units. When the rent is $600 per month, all 1000 units will be occupied. However, when the rent is $700 per month only 800 units will be occupied. Assume the relationship between the monthly rent and the number of units occupied is linear. (Create two points let the x-coordinates represent the price and the y coordinates the number of units rented.)

a) Write an equation for the number of units rented y in terms of the price p.
b) Use the equation to predict the number of units occupied when the price is $675. (round to the nearest unit if needed)
c) What price would cause 950 units to be occupied?
Section 4.2: Linear models – building linear functions from data

Don’t do any graphs in this section by hand. Just write see calculator for any questions that ask you to sketch a graph.

1. As Earth’s population continues to grow, the solid waste generated by the population grows with it. Governments must plan for disposal and recycling of ever growing amounts of solid waste. Planners can use data from the past to predict future waste generation and plan for enough facilities for disposing of and recycling the waste. (Don’t graph by hand. Write see Calculator for you answer.)

Given the following data on the waste generated in Florida from 1990-1994

<table>
<thead>
<tr>
<th>Year</th>
<th>Tons of Solid Waste Generated (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>19,358</td>
</tr>
<tr>
<td>1991</td>
<td>19,484</td>
</tr>
<tr>
<td>1992</td>
<td>20,293</td>
</tr>
<tr>
<td>1993</td>
<td>21,499</td>
</tr>
<tr>
<td>1994</td>
<td>23,561</td>
</tr>
</tbody>
</table>

a) Make a scatterplot of the data, letting x represent the number of years since 1990. Use x = 0 to represent 1990.
b) Use a graphing calculator to fit a linear function to the data.
c) Decide whether the new equation is a “good fit” to represent this data.
d) Graph the function of best fit with the scatterplot of the data.
e) Predict the average tons of waste in 2005

2. The numbers of insured commercial banks y (in thousands) in the United States for the years 1987 to 1996 are shown in the table. (Source: Federal Deposit Insurance Corporation).

|------|------|------|------|------|------|------|------|------|------|------|

a) Make a scatterplot of the data, letting x represent the number of years since 1980.
b) Use a graphing calculator to fit a linear function to the data.
c) Decide whether the new equation is a “good fit” to represent this data.
d) Graph the function of best fit with the scatterplot of the data.
e) Predict the average number of insured commercial banks in 2000.
3. **U.S. Farms.** As the number of farms has decreased in the United States, the average size of the remaining farms has grown larger, as shown in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Acreage Per Farm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1910</td>
<td>139</td>
</tr>
<tr>
<td>1920</td>
<td>149</td>
</tr>
<tr>
<td>1930</td>
<td>157</td>
</tr>
<tr>
<td>1940</td>
<td>175</td>
</tr>
<tr>
<td>1950</td>
<td>216</td>
</tr>
<tr>
<td>1959</td>
<td>303</td>
</tr>
<tr>
<td>1969</td>
<td>390</td>
</tr>
<tr>
<td>1978</td>
<td>449</td>
</tr>
<tr>
<td>1987</td>
<td>462</td>
</tr>
<tr>
<td>1997</td>
<td>487</td>
</tr>
</tbody>
</table>

a) Make a scatterplot of the data, letting $x$ represent the number of years since 1900.
b) Use a graphing calculator to fit a linear function to the data.
c) Decide whether the new equation is a "good fit" to represent this data.
d) Graph the function of best fit with the scatterplot of the data.
e) Predict the average acreage in 2010.

4. **Sports** The winning times (in minutes) in the women’s 400-meter freestyle swimming event in the Olympics from 1936 to 1996 are given by the following ordered pairs.

   (1936, 5.44) (1972, 4.32)
   (1948, 5.30) (1976, 4.16)
   (1952, 5.20) (1980, 4.15)
   (1956, 4.91) (1984, 4.12)
   (1960, 4.84) (1988, 4.06)
   (1964, 4.72) (1992, 4.12)
   (1968, 4.53) (1996, 4.12)

a) Make a scatterplot of the data, letting $x$ represent the number of years since 1936.
b) Use a graphing calculator to fit a linear function to the data.
c) Decide whether the new equation is a "good fit" to represent this data.
d) Graph the function of best fit with the scatterplot of the data.
e) Predict the winning time in 2020.
Section 4.2: Linear models – building linear functions from data

5) The average salary $S$ (in millions of dollars) for professional baseball players from 1996 to 2002 is shown in the table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary</td>
<td>1.1</td>
<td>1.3</td>
<td>1.4</td>
<td>1.6</td>
<td>1.8</td>
<td>2.1</td>
<td>2.3</td>
</tr>
</tbody>
</table>

a) Make a scatterplot of the data, letting $x$ represent the number of years since 1990.

b) Use a graphing calculator to fit a linear function to the data.

c) Decide whether the new equation is a "good fit" to represent this data.

d) Graph the function of best fit with the scatterplot of the data

e) Predict the average salary in 2018. (round to the nearest 10th)

6) The table show the number of Target stores worldwide from 1997 to 2002.

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Target stores</td>
<td>1130</td>
<td>1182</td>
<td>1243</td>
<td>1307</td>
<td>1381</td>
<td>1476</td>
</tr>
</tbody>
</table>

a) Make a scatterplot of the data, letting $x$ represent the number of years since 1990.

b) Use a graphing calculator to fit a linear function to the data.

c) Decide whether the new equation is a "good fit" to represent this data.

d) Graph the function of best fit with the scatterplot of the data

e) Predict the number of Target stores in 2020.
# Section 4.3: Quadratic functions and their properties

#1-12: For each problem do the following

a) Find the indicated function and describe the transformation as compared to the function \( f(x) = x^2 \), specifically state if the graph is shifted left, right, up, down and if any reflection has occurred

b) make a table of values and sketch a graph.

c) state the domain and range of the function

d) state the intervals where the function in increasing and decreasing

e) state if the function has a local maximum, if it does state the local maximum value

f) state if the function has a local minimum, if it does state the local minimum value

1) \( f(x - 3) + 4 \) 
2) \( f(x - 2) + 6 \) 
3) \( 2f(x + 3) - 4 \)

4) \( 3f(x + 1) + 2 \) 
5) \( \frac{1}{2} f(x + 4) - 6 \) 
6) \( \frac{1}{3} f(x + 3) - 1 \)

7) \( -2f(x) + 3 \) 
8) \( -3f(x) + 6 \) 
9) \( -\frac{1}{4} f(x + 5) - 2 \)

10) \( -\frac{1}{2} f(x + 2) - 1 \) 
11) \( 2f(x + 3) + 4 \) 
12) \( 2f(x + 1) + 5 \)

#13 – 24: For each problem do the following:

a) Use completing the square to rewrite the problem in standard form

b) Describe the transformation as compared to the function \( f(x) = x^2 \)

c) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph

13) \( f(x) = x^2 + 6x + 5 \) 
14) \( g(x) = x^2 + 10x - 11 \) 
15) \( k(x) = x^2 - 4x + 2 \)

16) \( m(x) = x^2 - 2x + 6 \) 
17) \( f(x) = 2x^2 + 8x - 3 \) 
18) \( h(x) = 4x^2 + 24x + 30 \)

19) \( f(x) = -x^2 + 6x + 4 \) 
20) \( g(x) = -x^2 - 8x - 2 \) 
21) \( k(x) = -2x^2 + 12x - 7 \)

22) \( g(x) = -2x^2 + 8x + 3 \) 
23) \( f(x) = -3x^2 - 12x + 1 \) 
24) \( n(x) = -2x^2 + 20x - 45 \)
Section 4.3: Quadratic functions and their properties.

#25 – 32, determine the equation of the quadratic function
Section 4.3: Quadratic functions and their properties.

29) \( (2, 0) \) \( (1, 4) \)

30) \( (2, -4) \) \( (4, 5) \)

31) \( (2, 3) \)

32) \( (2, 3) \) \( (8, 7) \)
Section 4.4: applications of quadratic equations

1) When a ball is thrown straight upward into the air, the equation \( h = -16t^2 + 80t \) gives the height \( h \) in feet that the ball is above the ground \( t \) seconds after it is thrown.
   a) How long does it take for the ball to hit the ground?
   b) When does the ball reach its maximum height?
   c) What is the maximum height of the ball?

2) An object fired vertically into the air it will be at a height \( h \) in feet, \( t \) seconds after launching, determined by the equation \( h = 96t - 16t^2 \).
   a) At which times will the object have a height of 80 feet?
   b) How long it will take the object to return to the ground.
   c) Determine the maximum height the object reaches.

3) A ball is shot into the air. Its height, \( h \) in meters after \( t \) seconds is modeled by \( h = -4.9t^2 + 30t + 1.6 \).
   a) How long (round to 2 decimal places) will it take the ball to reach a height of 35m?
   b) How long will it take to land (round to 2 decimals)?
   c) Determine the maximum height it reaches (round to 2 decimals).

4) A student tosses a water balloon of the roof of an apartment building. The height \( h \) in feet of the balloon, \( t \) seconds after it is launched, is given by the formula \( h = -16t^2 + 48t + 112 \). After how many seconds will the balloon hit the ground?

5) A baby drops his bottle at the peak of a ferris wheel. The height \( h \) in feet of the bottle, \( t \) seconds after the baby drops the bottle is given by \( h = -16t^2 + 64t + 80 \). After how many seconds will the bottle hit the ground?

6) A diver jumps off a cliff to water that is 240 feet below. The diver’s height \( h \) in feet \( t \) seconds after diving is given by \( h = -16t^2 + 240 \). How long does the dive last?

7) A diver jumps off a cliff to water that is 160 feet below. The diver’s height \( h \) in feet \( t \) seconds after diving is given by \( h = -16t^2 + 160 \). How long does the dive last?
Section 4.4: Applications of quadratic equations

8) A chain store manager has been told by the main office that daily profit, $P$, is related to the number of clerks working that day, $x$, according to the equations $P = -25x^2 + 300x$.

a) What number of clerks will maximize the profit?

b) What is the maximum possible profit?

9) The total profit ( $p(x)$ ) in dollars for a company to manufacture and sell $x$ items per week is given by the function $p(x) = -x^2 + 50x$.

a) What number of units will maximize profit?

b) What is the maximum profit?

10) A factory produces lemon-scented widgets. You know that the more you produce the lower the unit cost is, to a point. As production levels increase so do the labor costs, storage costs, etc. An accountant has computed the cost for producing $x$ thousands of units per day can be approximated by the formula $C(x) = 0.04x^2 - 8.504x - 25302$.

a) Find the daily production level that minimizes cost.

b) What is the minimum cost?

11) A manufacturer of lighting fixtures has a daily production cost of $C(x) = 0.25x^2 - 10x + 800$. Where $x$ is the number of units produced.

a) how many fixtures should be produced each day to minimize cost?

b) What is the minimum cost?
Section 4.4: Applications of quadratic equations

12) In one study the efficiency of photosynthesis in an Antarctic species of grass was investigated. Table 1 below lists results for various temperatures. The temperature \( x \) is in degrees Celsius and the efficiency \( y \) is given as a percent. The purpose of the research was to determine the temperature at which photosynthesis is most efficient.

<table>
<thead>
<tr>
<th>( x(\degree C) )</th>
<th>-1.5</th>
<th>0</th>
<th>2.5</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>17</th>
<th>20</th>
<th>22</th>
<th>25</th>
<th>27</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(%) )</td>
<td>33</td>
<td>46</td>
<td>55</td>
<td>80</td>
<td>87</td>
<td>93</td>
<td>95</td>
<td>91</td>
<td>89</td>
<td>77</td>
<td>72</td>
<td>54</td>
<td>46</td>
<td>34</td>
</tr>
</tbody>
</table>

\[ a) \text{ Make a scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.} \]
\[ b) \text{ Use a graphing calculator to fit a linear function and a quadratic function to the data.} \]
\[ c) \text{ Decide which equation is the best to represent this data.} \]
\[ d) \text{ Graph the function of best fit with the scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.} \]
\[ e) \text{ Use the equation to determine the optimal temperature at which photosynthesis is most efficient.} \]

13) A baseball player hits a ball. The following data represents the height of the ball at different times.

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>5</td>
<td>50</td>
<td>70</td>
<td>10</td>
</tr>
</tbody>
</table>

\[ a) \text{ Make a scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.} \]
\[ b) \text{ Use a graphing calculator to fit a linear function and a quadratic function to the data.} \]
\[ c) \text{ Decide which equation is the best to represent this data.} \]
\[ d) \text{ Graph the function of best fit with the scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.} \]
\[ e) \text{ Use the equation to find the maximum height of the ball.} \]

14) The following data represent the birth rate (per 1000 women) for women whose age is \( x \), in 2007.

<table>
<thead>
<tr>
<th>age</th>
<th>12</th>
<th>17</th>
<th>22</th>
<th>27</th>
<th>32</th>
<th>37</th>
<th>42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Birth rate</td>
<td>1</td>
<td>42</td>
<td>106</td>
<td>117</td>
<td>100</td>
<td>48</td>
<td>10</td>
</tr>
</tbody>
</table>

\[ a) \text{ Make a scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.} \]
\[ b) \text{ Use a graphing calculator to fit a linear function and a quadratic function to the data.} \]
\[ c) \text{ Decide which equation is the best to represent this data.} \]
\[ d) \text{ Graph the function of best fit with the scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.} \]
\[ e) \text{ Predict the birth rate of 35 year old women.} \]
Section 4.4: Applications of quadratic equations

15) The concentration (in milligrams per liter) of a medication in a patient’s blood as time passes is given by the data in the following table:

<table>
<thead>
<tr>
<th>Time (Hours)</th>
<th>Concentration (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>78.1</td>
</tr>
<tr>
<td>1</td>
<td>99.8</td>
</tr>
<tr>
<td>1.5</td>
<td>84.4</td>
</tr>
<tr>
<td>2</td>
<td>50.1</td>
</tr>
<tr>
<td>2.5</td>
<td>15.6</td>
</tr>
</tbody>
</table>

a) Make a scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.
b) Use a graphing calculator to fit a linear function and a quadratic function to the data.
c) Decide which equation is the best to represent this data.
d) Graph the function of best fit with the scatterplot of the data, on your calculator. You do not need to make a copy of this on your paper.
e) What is the concentration of medicine after 1.75 hours?
Ch 4 practice test.

1) Suppose \( f(x) = -2x - 6 \) and \( g(x) = 5x + 20 \)
   a) Solve \( g(x) = 0 \)       b) Solve \( f(x) > 0 \)       c) Solve \( f(x) = g(x) \)       d) Solve \( g(x) < f(x) \)

2) A school district purchases new televisions, one for each classroom for $25,000. After 8 years the televisions will be worth $1,000 and they will have to be replaced. Assume the relationship between the value of the televisions and the number of years in use is linear. Create two points in the form \( (x = \text{age of the televisions}, y = \text{value}) \)
   a) Write a linear function giving the value of the equipment for the 8 years it will be in use.
   b) Find the value of the televisions after 4 years.

3) A real estate office manages an apartment complex with 1000 units. When the rent is $900 per month, all 1000 units will be occupied. However, when the rent is $1100 per month only 900 units will be occupied. Assume the relationship between the monthly rent and the number of units occupied is linear.
   a) Write the equation giving the demand \( y \) in terms of the price \( p \)
   b) Use the equation to predict the number of units occupied when the price is $1400. (round to the nearest unit if needed)
   c) What price would cause 700 units to be occupied?

4) \( f(x) = x^2 \)
   a) Find \( f(x - 3) + 2 \)
   b) Describe the transformation as compared to the function \( f(x) = x^2 \), specifically state if the graph is shifted left, right, up, down and if any reflection has occurred
   c) make a table of values and sketch a graph
   d) state the domain and range of the function
   e) state the intervals where the function is increasing and decreasing

5) \( m(x) = x^2 - 6x + 5 \)
   a) Use completing the square to rewrite the problem in standard form
   b) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph

6) When a ball is thrown straight upward into the air, the equation \( h = -16t^2 + 20t \) gives the height (h) in feet that the ball is above the ground \( t \) seconds after it is thrown. (window x min 0 x max 10 y min 0 y max 20)
   a) How long does it take for the ball to hit the ground?
   b) When does the ball reach its maximum height? (expect a fraction answer)
   c) What is the maximum height of the ball?

7) A baseball player hits a ball. The following data represents the height of the ball at different times.

<table>
<thead>
<tr>
<th>Time (in seconds)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>12</td>
<td>19</td>
<td>27</td>
<td>28</td>
</tr>
</tbody>
</table>

   a) Use a graphing calculator to fit a quadratic function to the data. (round each number to 2 decimals) (window x min 0 x max 10 y min 0 y max 50)
   b) Use the equation to find the maximum height of the ball. (round to 2 decimals)