Section 5.1 Polynomial functions

#1 – 16
a) List each x-intercept (zero) and its multiplicity (round to 2 decimal places when needed)
b) Determine whether the graph crosses or touches the x-axis at each x-intercept
c) Determine the maximum number of turning points on the graph
d) Sketch a graph and approximate the turning points, also label the x-intercepts
e) Describe the end behavior
f) State the intervals where the function is increasing and decreasing

1) \( f(x) = (x - 3)^2(x + 1) \)  
2) \( f(x) = (x - 5)^2(x + 2) \)

3) \( f(x) = (x - 3)^3(x + 4) \)  
4) \( f(x) = (x - 2)^3(x + 1) \)

5) \( f(x) = (x + 3)(x - 3)(x + 6) \)  
6) \( f(x) = (x + 2)(x - 2)(x + 6) \)

7) \( f(x) = (x + 3)^2(x - 2) \)  
8) \( f(x) = (x + 4)^2(x - 3) \)

9) \( f(x) = x^2 + 6x - 7 \)  
10) \( f(x) = x^2 - 6x - 7 \)

11) \( f(x) = x^2 - 4 \)  
12) \( f(x) = x^2 - 9 \)

13) \( f(x) = x^3 - 6x^2 + 5x \)  
14) \( f(x) = 6x^3 - 4x^2 - 10x \)

15) \( f(x) = -3x^4 + 12x^2 \)  
16) \( f(x) = 2x^4 - 18x^2 \)

#17 – 26: Form a polynomial with whose x-intercepts are given. (Multiply out your polynomial)

17) x-intercepts: \((3,0), (-4,0)\); degree 2
18) x-intercepts \((-2,0), (4,0)\); degree 2

19) x-intercepts: \((3,0), \left(\frac{1}{2}, 0\right)\); degree 2
20) x-intercepts \((4,0), \left(\frac{2}{3}, 0\right)\); degree 2

21) x-intercepts: \((-3,0), \left(\frac{3}{4}, 0\right), (3,0)\); degree 3
22) x-intercepts \((-2,0), (2,0), \left(\frac{3}{2}, 0\right)\); degree 3

23) x-intercepts \((\sqrt{2}, 0), (-\sqrt{2}, 0)\); degree 2
24) x-intercepts \((\sqrt{3}, 0), (-\sqrt{3}, 0)\); degree 2

25) x-intercepts \((\sqrt{2}, 0), (-\sqrt{2}, 0), (6,0)\); degree 3
26) x-intercepts \((\sqrt{3}, 0), (-\sqrt{3}, 0), (7,0)\); degree 3
Section 5.2: synthetic division

#1-10:

a) Perform the division using synthetic division.
b) If the remainder is 0 use the result to completely factor the dividend (this is the numerator or the polynomial to the left of the division sign.)

1) \[
\frac{3x^3 - 17x^2 + 15x - 25}{x - 5}
\]
2) \[
\frac{5x^3 + 18x^2 + 7x - 6}{x + 3}
\]
3) \[
\frac{4x^3 + 8x^2 - 9x - 18}{x + 2}
\]
4) \[
\frac{9x^3 - 18x^2 - 16x + 32}{x - 2}
\]
5) \[
\frac{3x^3 - 16x^2 - 72}{x - 6}
\]
6) \[
\frac{5x^3 - 6x^2 + 8}{x - 4}
\]
7) \[(5x^3 + 6x + 8) ÷ (x + 2)
\]
8) \[(x^3 + 512) ÷ (x + 8)
\]
9) \[(x^3 - 27) ÷ (x - 3)
\]
10) \[(x^3 + 5x^2) ÷ (x + 5)
\]

#11-20:

a) Use your graphing calculator, or the rational root theorem to find a x-intercept of the polynomial
i) you need to find one x-intercept for a third degree polynomial
ii) you need to find two x-intercepts for a fourth degree polynomial
b) Use synthetic division to completely factor the polynomial (use “double” synthetic division for fourth degree polynomials)
c) Use your answer to part a to solve \( f(x) = 0 \)

11) \( f(x) = x^3 + 2x^2 - 5x - 6 \)
12) \( f(x) = x^3 + 8x^2 + 11x - 20 \)
13) \( f(x) = 2x^3 - 13x^2 + 24x - 9 \)
14) \( f(x) = 2x^3 - 5x^2 - 4x + 12 \)
15) \( f(x) = x^3 - 5x^2 - 4x + 20 \)
16) \( f(x) = x^3 - 3x^2 - 4x + 12 \)
17) \( f(x) = 2x^4 + 17x^3 + 35x^2 - 9x - 45 \)
18) \( f(x) = 4x^4 - 15x^3 - 8x^2 + 15x + 4 \)
19) \( f(x) = x^4 + 7x^2 - 8 \)
20) \( f(x) = x^4 - 25x^2 + 144 \)
Section 5.3:

#1 – 12: Create a function with lead coefficient 1 that satisfies the conditions.

1) degree 2; zeros 3i and -3i  
2) degree 2; zeros 4i and -4i  
3) degree 3; zeros -4 and 5i and -5i  
4) degree 3; zeros – 5 and 3i and -3i  
5) degree 3; zeros 2 and 3i  
6) degree 3; zeros 7 and 2i and -2i  
7) degree 4; zeros 2i, and 6i  
8) degree 4; zeros 3i and 4i  
9) degree 2; zero 3+i  
10) degree 2; zeros 4 + i  
11) degree 2; zero 5 - i  
12) degree 2; zero 6 - i

#13-16: Use synthetic division and the given zero to find the remaining zeros.

13) \( f(x) = x^3 - 4x^2 + 4x - 16 \) (x = 2i is a zero)  
14) \( f(x) = x^3 + 3x^2 + 25x + 75 \) (x = 5i is a zero)  
15) \( f(x) = x^3 - 4x^2 + 9x - 36 \) (x = 3i is a zero)  
16) \( f(x) = x^3 - 5x^2 + 9x - 45 \) (x = 3i is a zero)
Section 5.4: Properties of rational functions

#1-16:

a) find the domain, express your answer using words

b) find the equation of the vertical asymptote (if any)

1) \( f(x) = \frac{2x-6}{x^2+3} \)
2) \( f(x) = \frac{3x-12}{2x+6} \)
3) \( f(x) = \frac{3x+12}{x^2-3x-4} \)
4) \( f(x) = \frac{3x-18}{x^2+5x-6} \)

5) \( f(x) = \frac{3x^2-10x-8}{x^2-4} \)
6) \( f(x) = \frac{3x^2+20x+12}{x^2-16} \)
7) \( f(x) = \frac{x^2-16}{x^3-x^2} \)
8) \( f(x) = \frac{3x-18}{x^2+5x^2-6x} \)

9) \( f(x) = \frac{3}{x-6} \)
10) \( f(x) = \frac{2}{x-8} \)
11) \( f(x) = \frac{2x}{x^2-9} \)
12) \( f(x) = \frac{3x}{x^2-25} \)

13) \( f(x) = \frac{x+2}{x^2+16} \)
14) \( f(x) = \frac{3x-12}{x^2+25} \)
15) \( f(x) = \frac{5x}{x^2+9} \)
16) \( f(x) = \frac{2x}{x^2+1} \)

#1 7-32

a) find the x-intercept

b) find the y-intercept

17) \( f(x) = \frac{2x-6}{x+3} \)
18) \( f(x) = \frac{3x-12}{2x+6} \)

19) \( f(x) = \frac{3x+12}{x^2-3x-4} \)
20) \( f(x) = \frac{3x-18}{x^2+5x-6} \)

21) \( f(x) = \frac{3x^2-10x-8}{x^2-4} \)
22) \( f(x) = \frac{3x^2+20x+12}{x^2-16} \)

23) \( f(x) = \frac{x^2-16}{x^3-x^2} \)
24) \( f(x) = \frac{3x-18}{x^2+5x^2-6x} \)

25) \( f(x) = \frac{3}{x-6} \)
26) \( f(x) = \frac{2}{x-8} \)

27) \( f(x) = \frac{2x}{x^2-9} \)
28) \( f(x) = \frac{3x}{x^2-25} \)

29) \( f(x) = \frac{x+2}{x^2+16} \)
30) \( f(x) = \frac{3x-12}{x^2+25} \)

31) \( f(x) = \frac{5x}{x^2+9} \)
32) \( f(x) = \frac{2x}{x^2+1} \)
#33-48: find the equation of the horizontal asymptote

33) \( f(x) = \frac{2x-6}{x+3} \)  
34) \( f(x) = \frac{6x+12}{2x+4} \)

35) \( f(x) = \frac{3x+12}{x^2-3x-4} \)  
36) \( f(x) = \frac{3x-18}{x^2+5x-6} \)

37) \( f(x) = \frac{3x^2-10x-8}{x^2-4} \)  
38) \( f(x) = \frac{3x^2+20x+12}{x^2-16} \)

39) \( f(x) = \frac{x^2-16}{x^2-x} \)  
40) \( f(x) = \frac{5x^2-18}{5x^2-6x} \)

41) \( f(x) = \frac{3}{x-6} \)  
42) \( f(x) = \frac{2}{x-8} \)

43) \( f(x) = \frac{2x}{x^2-9} \)  
44) \( f(x) = \frac{3x}{x^2-25} \)

45) \( f(x) = \frac{x+2}{x^2+16} \)  
46) \( f(x) = \frac{3x-12}{x^2+25} \)

47) \( f(x) = \frac{5x^2}{x^2+9} \)  
48) \( f(x) = \frac{2x}{x^2+1} \)

#49 – 54: find the equation of the slant asymptote

49) \( f(x) = \frac{x^2+5x-6}{x+3} \)  
50) \( f(x) = \frac{x^2+6x-7}{x+1} \)

51) \( f(x) = \frac{x^2+7x+10}{x-2} \)  
52) \( f(x) = \frac{x^2+9x+18}{x-2} \)

53) \( f(x) = \frac{6x^2+5x+3}{2x-1} \)  
54) \( f(x) = \frac{3x^2+4x+10}{x-5} \)
Section 5.5: the graph of a rational function

For each problem find the following:
  a)  Domain
  b)  Vertical Asymptote (if any)
  c)  Horizontal asymptote, or slant asymptote
  d)  x-intercept(s) if any
  e)  y-intercept(s) if any
  f)  Sketch a graph of the function

1. \( f(x) = \frac{3x + 6}{x - 6} \)
2. \( f(x) = \frac{4x - 12}{2x + 2} \)
3. \( f(x) = \frac{4x^2 - 9}{x^2 - 1} \)
4. \( f(x) = \frac{9x^2 - 4}{x^2 - 1} \)
5. \( f(x) = \frac{3}{5x} \)
6. \( f(x) = \frac{4}{3x} \)
7. \( f(x) = \frac{x + 3}{x^2 - 16} \)
8. \( f(x) = \frac{x - 1}{x^2 + 4x - 12} \)
9) \( f(x) = \frac{x^2 + 5x - 6}{x + 3} \)
10) \( f(x) = \frac{x^2 + 6x - 7}{x + 1} \)
11) \( f(x) = \frac{x^2 + 7x + 10}{x - 2} \)
12) \( f(x) = \frac{x^2 + 9x + 18}{x - 2} \)
13) \( f(x) = \frac{3x - 12}{2x + 6} \)
14) \( f(x) = \frac{2x - 10}{x + 5} \)
15) \( f(x) = \frac{1}{x} \)
16) \( f(x) = \frac{3}{x} \)
17) \( f(x) = \frac{x + 5}{x^2} \)
18) \( f(x) = \frac{x + 4}{x^2} \)
Section 5.6: Quadratic and rational inequalities

#1 – 8: Use the graph of f(x) to solve

a) f(x) > 0
b) f(x) < 0

1) [Graph of a quadratic function with x-intercepts at (-1,0) and (3,0) and y-intercept at (0,10)]

2) [Graph of a quadratic function with x-intercepts at (-3,0) and (2,0) and y-intercept at (0,10)]

3) [Graph of a quadratic function with x-intercepts at (-4,0) and (3,0) and y-intercept at (0,12)]

4) [Graph of a quadratic function with x-intercepts at (-5,0) and (2,0) and y-intercept at (0,12)]
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5) 

6) 

7) 

8)
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#9 – 20: Use the graph of f(x) to solve

a) f(x) ≥ 0
b) f(x) ≤ 0

9) [Graph showing x-intercepts at (-5,0), (3,0), (7,0)]

10) [Graph showing x-intercepts at (-6,0), (2,0), (7,0)]

11) [Graph showing x-intercept at (2,0)]

12) [Graph showing x-intercept at (4,0)]
Section 5.6: Quadratic and rational inequalities

13) Graph showing a quadratic function with x-intercepts at (-5,0) and (3,0).

14) Graph showing a quadratic function with x-intercepts at (-3,0) and (4,0).

15) Graph showing a rational function with x-intercept at (4,0).

16) Graph showing a rational function with x-intercept at (3,0).
Section 5.6: Quadratic and rational inequalities

17) [Graph showing a quadratic function with a vertex at (3, 0)]

18) [Graph showing a quadratic function with a vertex at (3, 0)]

19) [Graph showing a rational function with a vertical asymptote at x = 3]

20) [Graph showing a rational function with a vertical asymptote at x = 3]
Section 5.6: Quadratic and rational inequalities

#21-32: Solve
a) \( f(x) > 0 \)
b) \( f(x) < 0 \)

21) \( f(x) = x^2 + 5x - 6 \)  
22) \( f(x) = x^2 + 4x - 5 \)

23) \( f(x) = 9 - x^2 \)  
24) \( f(x) = 16 - x^2 \)

25) \( f(x) = x^3 + 6x^2 - 9x - 54 \)  
26) \( f(x) = x^3 + 7x^2 - 4x - 28 \)
(Use synthetic division, or the table feature on your calculator to find the x-intercepts.)

27) \( f(x) = x^3 + 5x^2 - 6x \)  
28) \( f(x) = x^3 + 4x^2 - 5x \)

29) \( f(x) = \frac{x+3}{x-4} \)  
30) \( f(x) = \frac{x+5}{x-6} \)

31) \( f(x) = \frac{x-5}{x^2-4} \)  
32) \( f(x) = \frac{x-7}{x^2-9} \)

#33–42: Solve
a) \( f(x) \geq 0 \)
b) \( f(x) \leq 0 \)

33) \( f(x) = x^2 - 5x \)  
34) \( f(x) = x^2 - 3x \)

35) \( f(x) = (x - 2)^2 \)  
36) \( f(x) = (x - 4)^2 \)

37) \( f(x) = (x + 5)(x - 3)^2 \)  
38) \( f(x) = (x + 2)(x - 6)^2 \)

39) \( f(x) = \frac{x+2}{x-6} \)  
40) \( f(x) = \frac{x+4}{x-6} \)

41) \( f(x) = \frac{x+6}{x^2-9} \)  
42) \( f(x) = \frac{x+5}{x^2-4} \)
Chapter 5 review

For problems 1 – 3:
   a) List each x-intercept (zero) and its multiplicity (round to 2 decimal places when needed)
   b) Determine whether the graph crosses or touches the x-axis at each x-intercept
   c) Determine the maximum number of turning points on the graph
   d) sketch a graph and approximate the turning points, also label the x-intercepts
   e) Describe the end behavior
   f) state the intervals where the function is increasing and decreasing

1) \( f(x) = (x - 5)^2(x - 3) \)      2) \( f(x) = x^2 + 2x - 15 \)
3) \( f(x) = x^3 + 2x^2 - 8x \)

For problems 4 – 5:
   a) use your graphing calculator, or the rational root theorem to find a x-intercept of the polynomial
      i) you need to find one x-intercept for a third degree polynomial
      ii) you need to find two x-intercepts for a fourth degree polynomial
   b) use synthetic division to completely factor the polynomial (use “double” synthetic division for fourth degree polynomials)
   c) Solve \( f(x) = 0 \)

4) \( f(x) = x^3 + x^2 - 9x - 9 \)      5) \( f(x) = 4x^3 + 8x^2 - 9x - 18 \)

6) Form a polynomial with whose x-intercepts are given. (multiply out your polynomial)
   x-intercepts: (3,0), (-4,0); degree 2

7) Create a function with lead coefficient 1 that satisfies the conditions.
   degree 3: zeros – 5 and 4i
1) \( f(x) = x^3 + 2x^2 - 8x \)
   a) List each x-intercept (zero) and its multiplicity (round to 2 decimal places when needed)
   b) State whether the graph crosses or touches the x-axis at each x-intercept.
   c) Sketch a graph and approximate the turning points, also label the x-intercepts
   d) Describe the end behavior
   e) State the intervals where the function is increasing and decreasing (round to 2 decimals)

2) \( f(x) = 2x^3 - 3x^2 - 17x - 12 \)
   a) Use your graphing calculator, or the rational root theorem to find a zero of the polynomial
   b) Use synthetic division to completely factor the polynomial (test question often needs bottoms up factoring)
   c) Solve \( f(x) = 0 \)

Problem 3 has been deleted.

4) Create a function with lead coefficient 1 that satisfies the conditions;
   degree 3: zeros 6 and 5i

5) \( f(x) = x^2 + 3x - 10 \)
   Solve \( f(x) < 0 \)

6) \[ f(x) = \frac{2x + 8}{x - 2} \]
   a) Domain
   b) Vertical Asymptote (if any)
   c) Horizontal asymptote, or slant asymptote
   d) x-intercept(s) if any
   e) y-intercept(s) if any
   f) Sketch a graph of the function: label all the features found in parts b – e

7) \[ f(x) = \frac{x + 10}{x^2 + 4x - 5} \]
   a) Domain
   b) Vertical Asymptote (if any)
   c) Horizontal asymptote, or slant asymptote
   d) x-intercept(s) if any
   e) y-intercept(s) if any
   f) Sketch a graph of the function: label all the features found in parts b – e
1a) (-4,0) odd  (0,0) odd  (2,0) odd
1b) The graph will cross the x-axis at each x-intercept

1c)  

1d) falls to the left, rises to the right
1e) increasing \((−∞, −2.43) \cup (1.09, ∞)\) decreasing \((-2.43, 1.09)\)

2a) \(x = 4\) (also \(x = -1\))
2b) \(f(x) = (x-4)(x+1)(2x+3)\)
2c) \(x = 4, -1, -3/2\)

4) \(f(x) = x^3 - 6x^2 + 25x - 150\)

5) \(-5 < x < 2\)

6a) Domain all real numbers except 2
6b) \(x = 2\)
6c) \(y = 2\)
6d) \((-4,0)\)
6e) \((0, -4)\)
7a) Domain all real numbers except $x = -5, 1$
7b) $x = -5$ and $x = 1$
7c) $y = 0$
7d) $(0, 0)$
7e) $(0, -2)$
7f)