

Section 6.1: Composite functions

#1-10: Find the following

- a)  $(f \circ g)(x)$
- b) the domain of  $(f \circ g)(x)$
- c)  $(g \circ f)(x)$
- d) the domain of  $(g \circ f)(x)$

1)  $f(x) = 3x - 6$        $g(x) = 2x + 10$       2)  $f(x) = 4x - 5$        $g(x) = 5x - 7$

3)  $f(x) = x^2 + 5$        $g(x) = 3x - 4$       4)  $f(x) = x^2 + 2$        $g(x) = 5x - 1$

5)  $f(x) = x - 4$        $g(x) = x^2 + 2x - 1$       6)  $f(x) = x - 3$        $g(x) = x^2 - 4x - 2$

7)  $f(x) = \frac{2}{x+4}$        $g(x) = \frac{3}{x-7}$       8)  $f(x) = \frac{1}{x-6}$        $g(x) = \frac{1}{x+1}$

9)  $f(x) = \frac{1}{x-3}$        $g(x) = \frac{1}{x}$       10)  $f(x) = \frac{1}{x-5}$        $g(x) = \frac{1}{x}$

#11-14: Show that (This will be an important skill in section 6.2.)

- a)  $(f \circ g)(x) = x$
- b)  $(g \circ f)(x) = x$

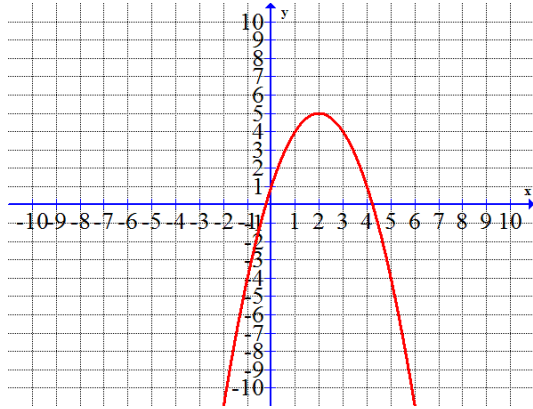
11)  $f(x) = 7x + 1$        $g(x) = \frac{x-1}{7}$       12)  $f(x) = 3 - 4x$        $g(x) = \frac{3-x}{4}$

13)  $f(x) = \frac{x-5}{2}$        $g(x) = 2x + 5$       14)  $f(x) = \frac{1}{x}$        $g(x) = \frac{1}{x}$

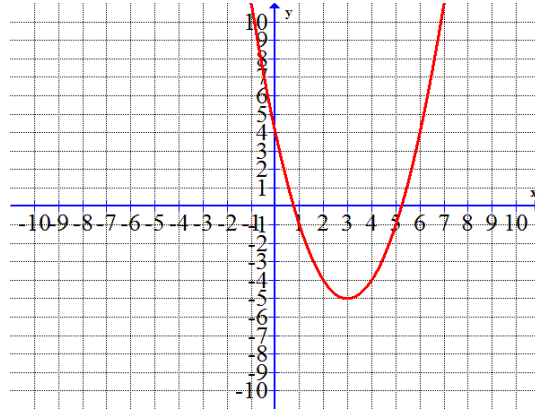
Section 6.2: One-to-one functions; inverse functions

#1 - 4: Determine if the functions are one to one by using the horizontal line test.

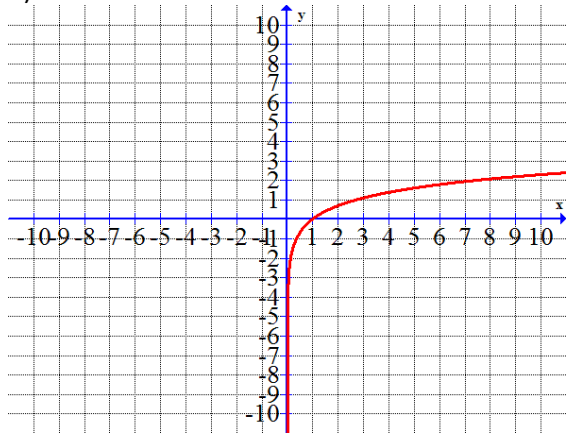
1)



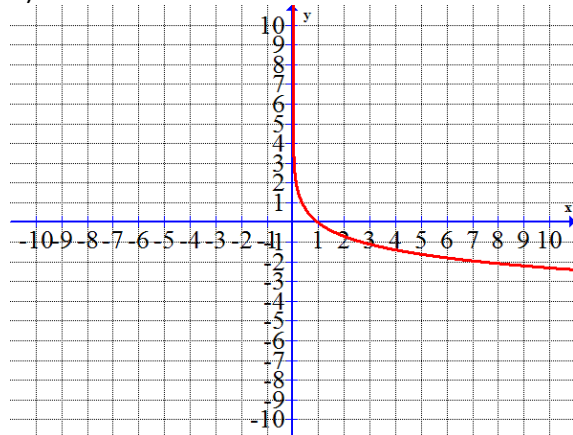
2)



3)



4)



#5 - 12: Use a graphing calculator to sketch a graph and determine whether the function is one to one or not.

5)  $f(x) = 2x - 5$

6)  $g(x) = x + 3$

7)  $f(x) = x^2 - 3$

8)  $k(x) = (x - 3)^2 + 6$

9)  $g(x) = x^4$

10)  $k(x) = x^4 - 3$

11)  $f(x) = x^3$

12)  $h(x) = x^3 - 2$

#13 - 18: Determine which of the functions are one to one. If a function is one to one find its inverse.

13)  $f = \{(0,1) (1,4) (2,4) (3,5)\}$

14)  $g = \{(3,2) (4,5) (-3,4) (1,5) (0,6)\}$

15)  $h = \{(0,3) (5,1) (7,11) (9, -3)\}$

16)  $k = \{(-3,4) (-5,6) (9, -3) (4, 0)\}$

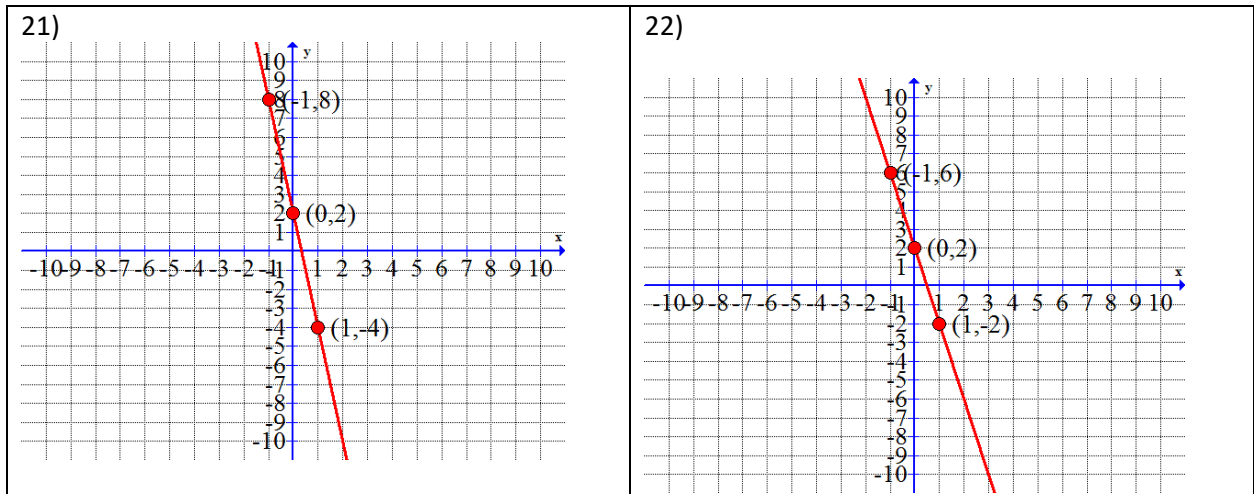
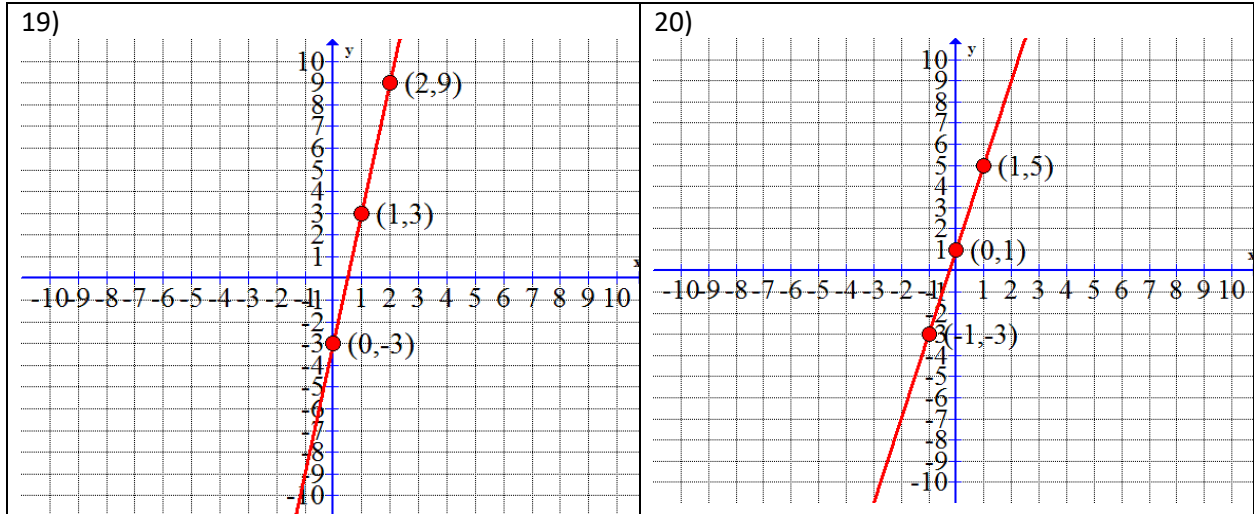
17)  $m = \{(0,2) (2,3) (3, 5)\}$

18)  $n = \{(1,1) (2,2) (3,4) (5,5)\}$

Section 6.2: One-to-one functions; inverse functions

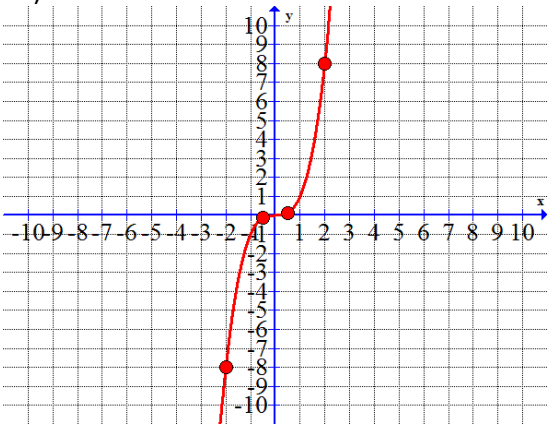
#19 – 26: The graph of a one to one function “ $f(x)$ ” is given.

Draw the graph of the inverse function  $f^{-1}(x)$



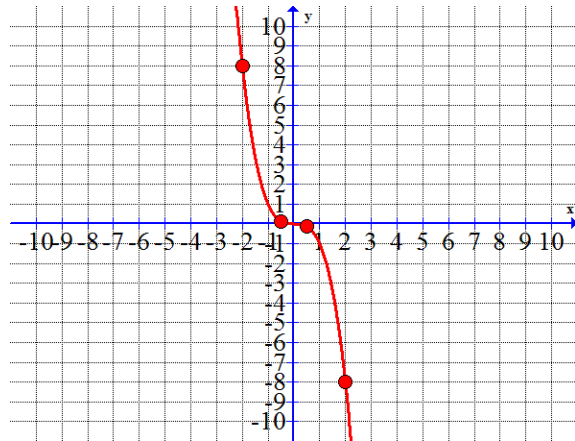
Section 6.2: One-to-one functions; inverse functions

23)



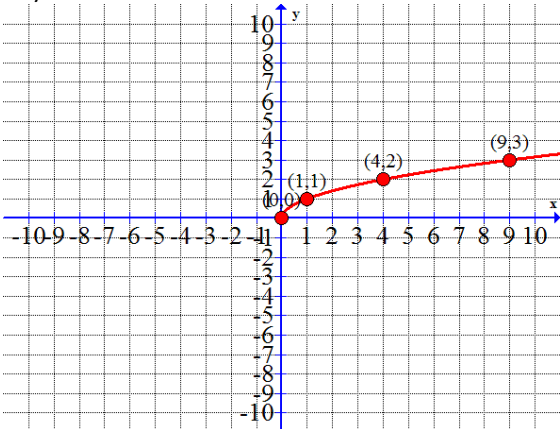
The points marked are  
 $(2, 8)$   $(1/2, 1/8)$   
 $(-1/2, -1/8)$   $(-2, -8)$

24)

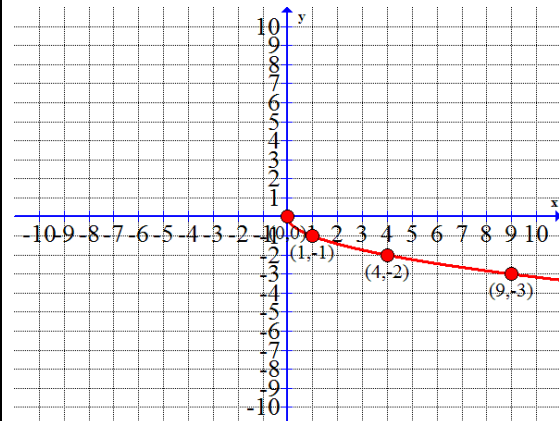


The points marked are  
 $(-2, 8)$   $(-1/2, 1/8)$   
 $(1/2, -1/8)$   $(2, -8)$

25)



26)



Section 6.2: One-to-one functions; inverse functions

#27 - 36: Each of the following functions is one to one. Perform the following:

- a) Find the inverse of each function, and express it using appropriate notation.
- b) Check your answer by showing that  $(f \circ f^{-1})(x) = x$  or  $(f^{-1} \circ f)(x) = x$
- c) Use your calculator to graph the function and its inverse and the line  $y = x$  on the same coordinate axis. This is just a visual test to confirm you have calculated the correct inverse.

27)  $f(x) = 2x - 4$

28)  $f(x) = 3x - 6$

29)  $f(x) = \frac{x-2}{3}$

30)  $f(x) = \frac{2x-5}{7}$

31)  $f(x) = \frac{2}{x}$

32)  $f(x) = \frac{3}{x}$

33)  $f(x) = \sqrt[3]{x-4}$

34)  $f(x) = \sqrt[3]{x-2}$

35)  $f(x) = x^3 + 2$

36)  $f(x) = x^3 - 3$

### Section 6.3: Exponential functions

1)  $f(x) = 2^x$

- a) Make a table of values and sketch a graph
- b) Find the domain of  $f(x)$
- c) Find the range of  $f(x)$
- d) Find the horizontal asymptote

2)  $f(x) = 3^x$

- a) Make a table of values and sketch a graph
- b) Find the domain of  $f(x)$
- c) Find the range of  $f(x)$
- d) Find the horizontal asymptote

3)  $f(x) = 2^{x+3}$

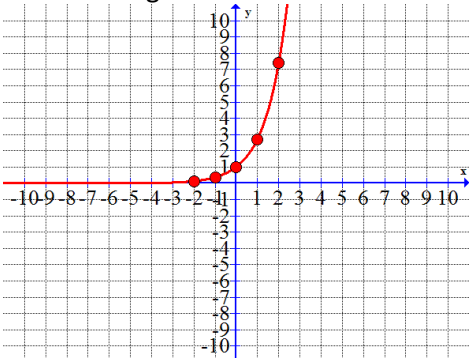
- a) Make a table of values and sketch a graph
- b) Find the domain of  $f(x)$
- c) Find the range of  $f(x)$
- d) Find the horizontal asymptote

4)  $f(x) = 3^{x+3}$

- a) Make a table of values and sketch a graph
- b) Find the domain of  $f(x)$
- c) Find the range of  $f(x)$
- d) Find the horizontal asymptote

# 5 – 16: Let  $f(x) = e^x$

Here is a graph of  $f(x) = e^x$  to help you complete the following #5 – 16.



The points plotted are as follows

| x  | f(x)                   |
|----|------------------------|
| -2 | $\frac{1}{e^2} = 0.14$ |
| -1 | $\frac{1}{e} = 0.37$   |
| 0  | 1                      |
| 1  | $e = 2.72$             |
| 2  | $e^2 = 7.39$           |

- Find the requested function
- Describe the transformation from  $f(x) = e^x$
- Use the transformation to sketch a graph
- Find the domain
- Find the range
- Find the horizontal asymptote

5)  $f(x-3)$

6)  $f(x-5)$

7)  $f(x+2)$

8)  $f(x+4)$

9)  $f(x) + 2$

10)  $f(x) + 3$

11)  $f(x) - 3$

12)  $f(x) - 4$

13)  $f(-x)$

14)  $-f(x)$

15)  $2f(x)$

16)  $3f(x)$

Section 6.3: Exponential functions

#17 - 32 Let  $g(x) = 2^x$

a) Find the indicated function

b) Describe the transformation of the graph of as compared to the graph of  $g(x) = 2^x$ .

17)  $g(x+1)$

18)  $g(x+3)$

19)  $g(x-1)$

20)  $g(x-4)$

21)  $g(x) + 1$

22)  $g(x) + 3$

23)  $g(x) - 2$

24)  $g(x) - 5$

25)  $-g(x)$

26)  $g(-x)$

27)  $g(x+1) - 4$

28)  $g(x + 2) - 3$

29)  $g(x - 2) + 3$

30)  $g(x - 3) + 1$

31)  $-g(-x) + 2$

32)  $-g(-x) + 1$

#33-50: Write each side with the same base then solve. Be sure to check your answer.

33)  $3^{x+2} = 27$

34)  $2^{x+5} = 16$

35)  $4^{-x} = 64$

36)  $6^{-x} = 216$

37)  $\left(\frac{1}{2}\right)^{x+3} = \frac{1}{32}$

38)  $\left(\frac{1}{3}\right)^{x-4} = \frac{1}{27}$

39)  $\left(\frac{1}{2}\right)^{2x+1} = 16$

40)  $\left(\frac{1}{2}\right)^{3x-4} = 32$

41)  $5^{x+2} = 25^{3x-4}$

42)  $2^{2x+1} = 8^{x-2}$

43)  $3^{x-4} = 9^{3-x}$

44)  $7^{4-5x} = 49^{x+1}$

45)  $e^{x-4} = e^{2x-3}$

46)  $e^{x-5} = e^{2x-4}$

47)  $e^{3x} * e^2 = e^{-4}$

48)  $e^{x-4} * e^{3x} = e^2$

49)  $2^{x-4} * 2^3 = 2^5$

50)  $3^{x-2} * 3^4 = 3^5$



## Section 6.4 Logarithmic Functions

#1 - 11: Write the equation in logarithmic form.

1)  $3^2 = 9$

2)  $2^3 = 8$

3)  $3^4 = 81$

4)  $2^5 = 32$

5)  $3^{-1} = \frac{1}{3}$

6)  $4^{-2} = \frac{1}{16}$

7)  $e^y = x$

8)  $e^0 = 1$

9)  $e^3 = 20.09$

10)  $e^2 = 7.39$

11)  $e^1 = 2.72$

#12 - 26: Write the equation in exponential form.

12)  $\log_2 8 = 3$

13)  $\log_3 81 = 4$

14)  $\log_2 16 = 4$

15)  $\log_2 64 = 6$

16)  $\log_5 1 = 0$

17)  $\log_6 6 = 1$

18)  $\log_3 x = 2$

19)  $\log x = 3$

20)  $\log x = 4$

21)  $\ln(x) = 1$

22)  $\ln(x) = 0$

23)  $\ln(2x) = w$

24)  $\ln(e) = 1$

25)  $\ln(e^2) = 2$

26)  $\ln(e^3) = 3$

#27 - 56: Find the logarithm value without using a calculator.

27)  $\log_2 2$

28)  $\log_2 1$

29)  $\log_7 7$

30)  $\log_3 3$

31)  $\log_3 1$

32)  $\log_3 9$

33)  $\log_4 1$

34)  $\log_4 4$

35)  $\log_4 64$

36)  $\log_5 1$

37)  $\log_5 5$

38)  $\log_5 125$

39)  $\log 1$

40)  $\log 10$

41)  $\log 100$

42)  $\log_2 32$

43)  $\log_2 128$

44)  $\log_3 243$

45)  $\log_2 2^3$

46)  $\log_3 3^4$

47)  $\log_5 5^6$

48)  $\log_7 7^8$

49)  $\log_4 4^5$

50)  $\log_2 2^9$

51)  $\ln(e)$

52)  $\ln(e^2)$

53)  $\ln(e^3)$

54)  $\ln(1)$

55)  $\ln(e^{-2})$

56)  $\ln(e^{-3})$

Section 6.4 Logarithmic Functions

#57 - 65: Use a calculator to approximate the logarithms. Round to 4 decimal places.

57)  $\log 6$

58)  $\log 5$

59)  $\log \frac{1}{3}$

60)  $\log \left(\frac{1}{120}\right)$

61)  $\log(5^{-3})$

62)  $\log(4^{-2})$

63)  $\ln(7)$

64)  $\ln(3)$

65)  $\ln(2^4)$

#66-71 (graphs of common logarithmic functions)

a) Graph the logarithmic functions. First write the equation in exponential form, then create a table of values and plot the points.

b) State the domain of each function.

66)  $y = \log_2 x$

67)  $y = \log_3 x$

68)  $y = \log_4 x$

69)  $y = \ln(x)$

70)  $y = \log_{1/2} x$

71)  $y = \log_{1/3} x$

#72- 83: Let  $f(x) = \log_2(x)$

| <p>Here is a graph of <math>f(x) = \log_2(x)</math> to help you out</p> | <p>Here are the points that are marked</p> <table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>.25</td> <td>-2</td> </tr> <tr> <td>.5</td> <td>-1</td> </tr> <tr> <td>1</td> <td>0</td> </tr> <tr> <td>2</td> <td>1</td> </tr> <tr> <td>4</td> <td>2</td> </tr> </tbody> </table> | x | f(x) | .25 | -2 | .5 | -1 | 1 | 0 | 2 | 1 | 4 | 2 |
|---|--|---|------|-----|----|----|----|---|---|---|---|---|---|
| x   | f(x)   |   |      |     |    |    |    |   |   |   |   |   |   |
| .25   | -2   |   |      |     |    |    |    |   |   |   |   |   |   |
| .5  | -1   |   |      |     |    |    |    |   |   |   |   |   |   |
| 1   | 0  |   |      |     |    |    |    |   |   |   |   |   |   |
| 2   | 1  |   |      |     |    |    |    |   |   |   |   |   |   |
| 4   | 2  |   |      |     |    |    |    |   |   |   |   |   |   |

a) Find the requested function.

b) State the domain of each function.

c) Describe the transformation that occurs from a common function

d) Graph the logarithmic functions

72)  $f(x + 2)$

73)  $f(x + 1)$

74)  $f(x - 3)$

75)  $f(x - 2)$

76)  $f(x) + 1$

77)  $f(x) + 2$

78)  $f(x) - 1$

79)  $f(x) - 2$

80)  $-f(x)$

81)  $f(-x)$

82)  $2f(x)$

83)  $3f(x)$

Section 6.5: Properties of Logarithms

#1 - 12: Evaluate each expression without a calculator. Check your answer using your calculator.

- |                 |                   |                    |
|-----------------|-------------------|--------------------|
| 1) $\log_2 16$  | 2) $\log_3 9^2$   | 3) $\log_4 16^3$   |
| 4) $\log_7 1$   | 5) $\log_8 8^5$   | 6) $\log_6 216$    |
| 7) $\log_3 243$ | 8) $\ln(e^4)$     | 9) $\ln(e)$        |
| 10) $\ln(1)$    | 11) $\log_2 64^3$ | 12) $\log_5(25^2)$ |

#13 – 24: Expand into sums and differences of logarithms (express exponents as multiplication).

- |                                |                                     |   |
|--------------------------------|-------------------------------------|---|
| 13) $\log_3(x^2y^3)$           | 14) $\log_4(xy^3z^5)$               | 15) $\log_5(x^2y^6z)$                     |
| 16) $\log_b \frac{x^4}{y}$     | 17) $\log_2 \frac{xy^3}{z^2}$       | 18) $\log_7 \frac{x}{yz^3}$               |
| 19) $\log_2 \frac{xy}{w^2z^5}$ | 20) $\log_3(x^2y)$                  | 21) $\log_4(x^3y^4)$                      |
| 22) $\log_4(y\sqrt{z})$        | 23) $\log_2(x^2 \cdot \sqrt[3]{y})$ | 24) $\log_5 \frac{\sqrt{x}}{\sqrt[3]{y}}$ |

#25 - 36: Write the expression as a single logarithm. Write your answer with only positive exponents.

- |                                   |  |  |
|-----------------------------------|--|--|
| 25) $3\log_2 x + 4\log_2 y$       | 26) $5\ln x + \ln y + 3\ln z$          | 27) $2\log_3 x + 4\log_3 y + \log_3 z$ |
| 28) $2\log x - 3\log y$           | 29) $5\log_2 x + 3\log_2 y - \log_2 z$ | 30) $4\log x + 2\log y - 3\log z$      |
| 31) $4\log x - 2\log y - 3\log z$ | 32) $2\ln x - 4\ln y + \ln z$          | 33) $-2\log_3 x + \log_3 y + \log_3 z$ |
| 34) $-\ln x + \ln y + \ln z$      | 35) $\ln x + 3\ln y - 2\ln z$          | 36) $4\ln x - 2\ln y + 5\ln z$         |

#37 - 45: Use the change of base formula and your calculator to evaluate each logarithm, round your answer to 2 decimal places.

- |                 |                   |                    |
|-----------------|-------------------|--------------------|
| 37) $\log_2 3$  | 38) $\log_7 14$   | 39) $\log_3 5$     |
| 40) $\log_9 36$ | 41) $\log_4 0.65$ | 42) $\log_2 0.25$  |
| 43) $\log_5 18$ | 44) $\log_6 7$    | 45) $\log_9 0.123$ |

#46 – 57: Find the following. Given  $\log_b x = 5$ ,  $\log_b y = 10$ ,  $\log_b z = 7$ .

- |                            |                               |                             |
|----------------------------|-------------------------------|-----------------------------|
| 46) $\log_b x + \log_b y$  | 47) $\log_b y + \log_b z$     | 48) $\log_b x^3$            |
| 49) $\log_b y^4$           | 50) $\log_b xy$               | 51) $\log_b yz$             |
| 52) $\log_b(x^2y^3)$       | 53) $\log_b(y^3z^5)$          | 54) $\log_b(x^2y^6z)$       |
| 55) $\log_b \frac{x^4}{y}$ | 56) $\log_b \frac{xy^3}{z^2}$ | 57) $\log_b \frac{x}{yz^3}$ |

Section 6.6: Logarithmic and exponential equations

#1 - 12: Solve the exponential equation by writing each side of the equation with the same base then equating the exponents. Problems also may be solved with logarithms.

1)  $2^x = 16$

2)  $3^x = 27$

3)  $2^{x+1} = 32$

4)  $3^{x+2} = 81$

5)  $\left(\frac{1}{2}\right)^x = 16$

6)  $\left(\frac{1}{3}\right)^x = 27$

7)  $2^{4-x} = 64$

8)  $3^{5-x} = 243$

9)  $32^x = 2$

10)  $27^x = 3$

11)  $16^x = 4$

12)  $49^x = 7$

#13 - 24: Solve the exponential equations, round your answer to 2 decimals.

13)  $3^x = 6$

14)  $2^x = 10$

15)  $e^x = 12$

16)  $e^x = 1.15$

17)  $5(10^x) = 20$

18)  $4(3^x) = 24$

19)  $32e^{2x} = 128$

20)  $14e^{3x} = 42$

21)  $3^{x-1} = 5^x$

22)  $2^x = 3^{x-1}$

23)  $6^{x-1} = 9^x$

24)  $2^x = 6^{x-3}$

#25 - 51: Solve the logarithmic equations, round to 2 decimals when needed.

25)  $\log_3 x = 2$

26)  $\log_2 x = 3$

27)  $\ln x = 1$

28)  $\ln x = 0$

29)  $\log_x 49 = 2$  ( $x > 0$ )

30)  $\log_x 27 = 3$  ( $x > 0$ )

31)  $\log_x 64 = 3$  ( $x > 0$ )

32)  $\log_x 32 = 5$  ( $x > 0$ )

33)  $\log_x 3 = \frac{1}{2}$  ( $x > 0$ )

34)  $\log_x 4 = \frac{1}{3}$  ( $x > 0$ )

35)  $\log_2 (x-1) = 3$

36)  $\log_3 (x-5) = 2$

37)  $\log_2 (2x) = 5$

38)  $\log_3 (3x) = 0$

39)  $\log (x+1) = \log (3x-2)$

40)  $\ln (x-4) = \ln (2x-10)$

41)  $\log_2 (x+3) = \log_2 (3x)$

42)  $\log_4 (3x+6) = \log_4 (4x)$

43)  $\log_2 x - \log_2 (x+6) = -2$

44)  $\log_3 x - \log_3 (x+6) = -1$

45)  $\log_2 x - \log_2 (x-6) = 2$

46)  $\log_3 (x-5) - \log_3 (x+3) = -2$

47)  $\log_2 (x+6) - \log_2 (3x+2) = -1$

48)  $\log_2 (x+2) + \log_2 x = 3$

49)  $\log_3 x + \log_3 (x+6) = 3$

50)  $\log_4 x + \log_4 (x+12) = 3$

51)  $\log_3 (x+6) + \log_3 (3x) = 4$

Section 6.7: financial models

#1-4: Use the compound interest formula  $A = P \left(1 + \frac{r}{n}\right)^{nt}$  to answer the following.

- 1) An initial deposit of \$1,000 earns 4% interest compounded twice per year. How much will be in the account after 5 years?
- 2) An initial deposit of \$1,000 earns 3% interest compounded monthly. How much will be in the account after 10 years?
- 3) An initial deposit of \$15,000 earns 2% interest compounded quarterly. How much will be in the account after 8 years?
- 4) An initial deposit of \$12,000 earns 1% interest compounded quarterly. How much will be in the account after 10 years?

#5-12: Use the formula  $A = Pe^{rt}$  to answer the following.

- 5) An initial investment of \$5,000 earns 6% interest compounded continuously. What will the investment be worth in 5 years?
- 6) An initial investment of \$10,000 earns 5.25% interest compounded continuously. What will the investment be worth in 8 years?
- 7) An initial investment of \$15,000 earns 3% interest compounded continuously. What will the investment be worth in 6 years?
- 8) An initial investment of \$120,000 earns 2.25% interest compounded continuously. What will the investment be worth in 22 years?
- 9) How long will it take an initial investment of \$1,000 to triple if it is expected to earn 6% interest compounded continuously? (Round to 1 decimal place)
- 10) How long will it take an initial investment of \$1,000 to double if it is expected to earn 6% interest compounded continuously? (Round to 1 decimal place)
- 11) How long will it take an initial investment of \$100,000 to grow to \$1,000,000 if it is expected to earn 4% interest compounded continuously? (Round to 1 decimal place)
- 12) How long will it take an initial investment of \$10,000 to grow to \$15,000 if it is expected to earn 4% interest compounded continuously? (Round to 1 decimal place)

Section 6.7: financial models

#13-18: Use the compound interest formula  $A = P \left(1 + \frac{r}{n}\right)^{nt}$  to answer the following.

- 13) What will a \$200,000 home cost in in 5 years if the price appreciation over that period is expected to be 3% compounded annually? (round to 2 decimals)
- 14) What will a \$150,000 home cost in in 10 years if the price appreciation over that period is expected to be 2% compounded annually? (round to 2 decimals)
- 15) What will a \$100,000 home cost in in 3 years if the price appreciation over that period is expected to be 2% compounded annually? (round to 2 decimals)
- 16) What will a \$300,000 home cost in in 20 years if the price appreciation over that period is expected to be 2.5% compounded annually? (round to 2 decimals)
- 17) The tuition at a local community college is \$80 per credit hour. Tuition is expected to increase at a rate of 4% per compounded annually. What will the tuition be in 5 years? (round to 2 decimals)
- 18) The tuition at a local community college is \$60 per credit hour. Tuition is expected to increase at a rate of 3% per compounded annually. What will the tuition be in 10 years? (round to 2 decimals)

## Section 6.8: Exponential Growth and Decay

1) The number of people who have heard a rumor can be modeled by the function  $n(d) = 8e^{.22d}$   $0 \leq d \leq 30$  (where  $n(d)$  represents the number of people who have heard the rumor after  $d$  days.)

1a) Determine the number of people who have heard the rumor after 4 days. (Round to the nearest person.)

1b) What is the growth rate of the number of people that have heard the rumor?

1c) How many days will it take until 1571 people have heard the rumor? (Round to the nearest day.)

2) The number of people who have heard a rumor can be modeled by the function  $n(d) = 5e^{.25d}$   $0 \leq d \leq 20$  (where  $n(d)$  represents the number of people who have heard the rumor after  $d$  days.)

2a) Determine the number of people who have heard the rumor after 5 days. (Round to the nearest person.)

2b) What is the growth rate of the number of people that have heard the rumor?

2c) How many days will it take until 100 people have heard the rumor? (Round to the nearest day.)

3) The size  $P(t)$  of a certain insect population in time  $t$  (in days) obeys the model  $P(t) = 200e^{.04t}$

3a) Determine the number of insects at  $t = 0$  days. (This is called the initial population.)

3b) What is the growth rate of the insect population?

3c) What will the population be after 7 days? (round to the nearest whole number)

3d) When will the population reach 3,000 insects? (round to 2 decimals)

3e) How long will it take for the population to double? (round to 2 decimals)

4) The size  $P(t)$  of a certain insect population in time  $t$  (in days) obeys the model  $P(t) = 500e^{.06t}$

4a) Determine the number of insects at  $t = 0$  days. (This is called the initial population.)

4b) What is the growth rate of the insect population?

4c) What will the population be after 9 days? (round to the nearest whole number)

4d) When will the population reach 10,000 insects? (round to 2 decimals)

4e) How long will it take for the population to triple? (round to 2 decimals)

5) Krypton-85 is a radioactive material that decays according to the function  $P(t) = P_0e^{-.063t}$  where  $P_0$  is the initial amount present and  $P(t)$  is the amount present after  $t$  years. Assume a scientist has a sample of 1000 grams of Krypton-85.

5a) What is the decay rate of Krypton-85 ?

5b) How much Krypton-85 will be left after 30 years? (round to 2 decimals)

5c) When will 200 grams of Krypton-85 be left? (round to 2 decimals)

5d) What is the half-life of Krypton-85 ? (round to 2 decimals)

## Section 6.8: Exponential Growth and Decay

6) Caffeine leaves the human body decays according to the function  $P(t) = P_0e^{-.139t}$ , where  $P_0$  is the initial amount present and  $P(t)$  is the amount present at time  $t$  in hours. Assume that a person has just consumed 100 grams of caffeine.

6a) What is the decay rate of caffeine?

6b) How much caffeine will be left after 21 hours? (round to 2 decimals)

6c) When will 10 grams of caffeine be left? (round to 2 decimals)

6d) What is the half-life of caffeine? (round to 2 decimals)

#7-10: Use the formula  $P(t) = P_0e^{kt}$ , where  $P_0$  is the initial population at  $t=0$ , and  $k$  is the rate of growth. (Round  $k$  to 3 decimals.)

7) The rodent population in a given city rose from 20,000 to 30,000 in 1 year. How long will it take the population to reach 100,000? (round to 2 decimals)

8) The bacterial in a laboratory culture increased from an initial population of 500 to 1,500 in 3 hours. How long will it take the population to reach 10,000? (hint first use the 500 to 1,500 in 3 hours and the formula above to solve for  $k$ , then use the 10,000 to answer the question.) (round to 2 decimals)

9) The population of a city is expected to triple in 20 years. The city currently has 10,000 residents. How long will it take to get to 50,000 residents? (round to 2 decimals)

10) The population of a city is expected to double in 20 years. The city currently has 1,000 residents. How long will it take to get to 5,000 residents? (round to 2 decimals)

#11-14- Use the formula: , where  $P(t) = P_0e^{kt}$  is the amount present at time  $t = 0$ , (Round  $k$  to 3 decimals)

11) The half life of a certain element is 10 years. A 20 ounce sample of the element is obtained. How long will it take for there to be 12 ounces of the sample left? (round to 2 decimals)

12) The half life of a certain element is 100 years. A 40 ounce sample of the element is obtained. How long will it take until 10 ounces of the sample are left? (round to 2 decimals)

13) The half life of a certain element is 5 years. A 60 ounce sample of the element is obtained. How long will it take until 15 ounces of the sample are left? (round to 2 decimals)

14) The half life of a certain element is 100 years. A 10 ounce sample of the element is obtained. How long will it take until 2 ounces of the sample are left? (round to 2 decimals)



## Section 6.9: Building Exponential and Logarithmic models

1) The data shows the cooling temperatures of a freshly brewed cup of coffee after it is poured from the brewing pot into a serving cup. The brewing pot temperature is approximately 180° F.

|                          |       |     |     |       |       |       |       |       |     |       |
|--------------------------|-------|-----|-----|-------|-------|-------|-------|-------|-----|-------|
| Time (minutes)           | 0     | 5   | 8   | 11    | 15    | 18    | 22    | 25    | 30  | 34    |
| Temperature (Fahrenheit) | 179.5 | 164 | 154 | 149.2 | 141.7 | 134.6 | 125.4 | 123.5 | 120 | 118.5 |

1a) Plot the data using a graphing calculator

1b) Decide the type of regression model that is appropriate

1c) Find the appropriate regression equation, and write it in the form  $P(t) = P_0e^{kt}$

1d) Predict when the coffee temperature will be 115 degrees Fahrenheit.

2) The data shows the cooling temperatures of a car engine after the engine is shut off.

|                          |     |     |     |     |     |     |     |     |     |     |
|--------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Time (minutes)           | 0   | 5   | 10  | 15  | 20  | 25  | 30  | 35  | 40  | 45  |
| Temperature (Fahrenheit) | 240 | 220 | 202 | 186 | 172 | 159 | 147 | 140 | 136 | 133 |

2a) Plot the data using a graphing calculator

2b) Decide the type of regression model that is appropriate

2c) Find the appropriate regression equation, and write it in the form  $P(t) = P_0e^{kt}$

2d) Predict when the engines temperature will be 140 degrees Fahrenheit.

Section 6.9: Building Exponential and Logarithmic models

3) The following is a table comparing the number of people who got sick after exposure to a certain virus.

|                       |    |    |    |     |     |     |
|-----------------------|----|----|----|-----|-----|-----|
| Days after exposure   | 1  | 2  | 3  | 4   | 5   | 6   |
| Number of people sick | 21 | 45 | 96 | 208 | 450 | 971 |

3a) Plot the data using a graphing calculator

3b) Decide the type of regression model that is appropriate

3c) Find the appropriate regression equation, and write it in the form  $P(t) = P_0e^{kt}$

3d) Predict when 2000 people will be sick

4) The following is a table comparing the number computers infected with a virus.

|                                    |    |    |    |     |     |     |
|------------------------------------|----|----|----|-----|-----|-----|
| Days after virus creation          | 1  | 2  | 3  | 4   | 5   | 6   |
| Number of computers with the virus | 10 | 39 | 82 | 163 | 312 | 656 |

4a) Plot the data using a graphing calculator

4b) Decide the type of regression model that is appropriate

4c) Find the appropriate regression equation, and write it in the form  $P(t) = P_0e^{kt}$

4d) Predict when 1 million computers will have the virus.

5) A chemist has a 50-gram sample of a radioactive material. He records the amount of radioactive material present every week for 6 weeks and obtains the following data

|                |    |    |    |    |    |    |    |
|----------------|----|----|----|----|----|----|----|
| week           | 0  | 1  | 2  | 3  | 4  | 5  | 6  |
| Weight (grams) | 50 | 44 | 38 | 33 | 29 | 26 | 23 |

5a) Plot the data using a graphing calculator

5b) Decide the type of regression model that is appropriate

5c) Find the appropriate regression equation, and write it in the form  $P(t) = P_0e^{kt}$

5d) Predict when there will be 10 grams of the material left.

Section 6.9: Building Exponential and Logarithmic models

6) A chemist has a 1000-gram sample of a radioactive material. He records the amount of radioactive material present every week for 6 weeks and obtains the following data

|                |      |     |     |     |     |     |     |
|----------------|------|-----|-----|-----|-----|-----|-----|
| week           | 0    | 1   | 2   | 3   | 4   | 5   | 6   |
| Weight (grams) | 1000 | 960 | 922 | 885 | 850 | 815 | 783 |

- 6a) Plot the data using a graphing calculator
- 6b) Decide the type of regression model that is appropriate
- 6c) Find the appropriate regression equation, and write it in the form  $P(t) = P_0e^{kt}$
- 6d) Predict when there will be 500 grams of the material left.

7) The data below show the average growth rates of 12 Weeping Higan cherry trees planted in Washington, D.C. At the time of planting, the trees were one year old and were all 6 feet in height.

|                      |   |     |    |    |      |      |      |    |      |      |
|----------------------|---|-----|----|----|------|------|------|----|------|------|
| Age of tree in years | 1 | 2   | 3  | 4  | 5    | 6    | 7    | 8  | 9    | 10   |
| height in feet       | 6 | 9.5 | 13 | 15 | 16.7 | 17.5 | 18.5 | 19 | 19.5 | 19.7 |

- 7a) Plot the data using a graphing calculator
- 7b) Decide the type of regression model that is appropriate
- 7c) Find the appropriate regression equation, (the equation your calculator generates will not need to be changed)
- 7d) Predict when the trees will be 20 feet in height.

8) The data below show the average growth rates of a Chilean Mesquite tree. At the time of planting, the trees was one year old and 4 feet tall.

|                      |   |     |    |      |      |    |    |      |       |      |
|----------------------|---|-----|----|------|------|----|----|------|-------|------|
| Age of tree in years | 1 | 2   | 3  | 4    | 5    | 6  | 7  | 8    | 9     | 10   |
| height in feet       | 4 | 7.5 | 10 | 12.5 | 14.5 | 16 | 17 | 17.5 | 17.75 | 17.9 |

- 8a) Plot the data using a graphing calculator
- 8b) Decide the type of regression model that is appropriate
- 8c) Find the appropriate regression equation, (the equation your calculator generates will not need to be changed)
- 8d) Predict when the trees will be 19 feet in height

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Chapter 6 Practice Test Part 1

1)  $f(x) = x^2 - 2x + 1$        $g(x) = 7x - 5$

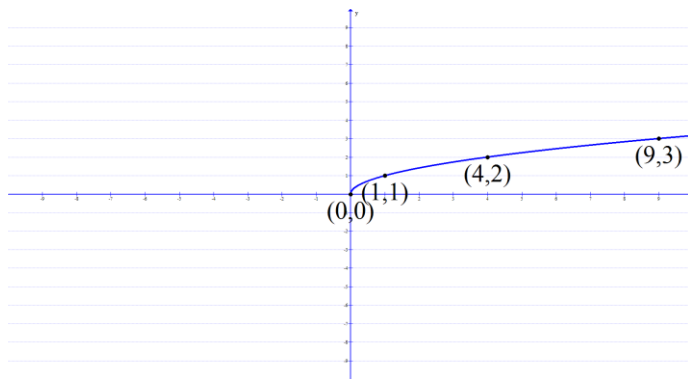
*Hint for part a:* This is the same as  $g(f(x))$  and it requires replacing any  $x$  in the right side of the  $g$ -function with the entire right side of the  $f$ -function.

*Hint for part b:* Since both  $g$  and  $f$  are polynomials there will be no algebra required to find this domain. In fact if you do any algebra you will likely find an  $x$ -intercept and it will not belong as part of a correct answer.

a)  $(g \circ f)(x)$

b) the domain of  $(g \circ f)(x)$

2) The graph of a one to one  $f$  function is given. Draw the graph of the inverse function  $f^{-1}$



3)  $f(x) = x^3 + 4$

a) Find the inverse of  $f(x)$

b) Check your answer by showing that  $(f \circ f^{-1})(x) = x$

*Steps to find inverse.*

i) Change from function notation to  $x$   $y$  notation.

ii) Switch  $x$  and  $y$

iii) solve for  $y$

iv) replace  $y$  with  $f^{-1}(x)$

v) move on to part b and check your answer

4)  $f(x) = e^x$

Find the requested function and describe the transformation compared to  $f(x)$

(Hint:  $f(x + h)$  shifts left  $h$        $f(x - h)$  shifts right  $h$        $f(x) + h$  shifts up  $h$        $f(x) - h$  shifts down  $h$   
 $-f(x)$  reflects over  $x$ -axis       $f(-x)$  reflects over  $y$ -axis)

a)  $f(x - 2)$

b)  $f(x) + 4$

c)  $-f(x)$

d)  $f(x + 3) - 2$

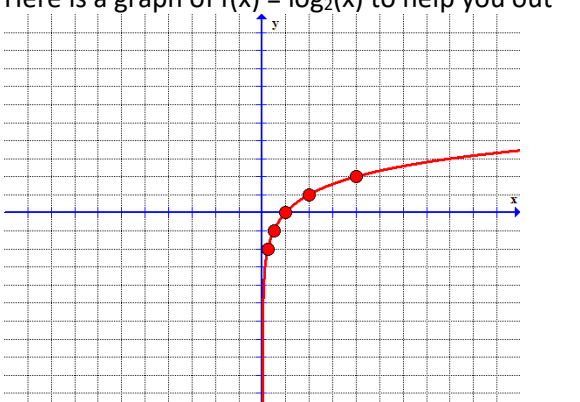
#5-7: Solve

5)  $3^{x+2} = 81$

6)  $\left(\frac{1}{2}\right)^{x+1} = \frac{1}{16}$

7)  $4^{3x+1} \cdot 4^{2x-3} = 4^{18}$

8 - 9: Let  $f(x) = \log_2(x)$

| <p>Here is a graph of <math>f(x) = \log_2(x)</math> to help you out</p>  | <p>Here are the points that are marked</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">x</th> <th style="padding: 5px;">f(x)</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">.25</td> <td style="padding: 5px;">-2</td> </tr> <tr> <td style="padding: 5px;">.5</td> <td style="padding: 5px;">-1</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">2</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">4</td> <td style="padding: 5px;">2</td> </tr> </tbody> </table> | x | f(x) | .25 | -2 | .5 | -1 | 1 | 0 | 2 | 1 | 4 | 2 |
|--|--|---|------|-----|----|----|----|---|---|---|---|---|---|
| x  | f(x)   |   |      |     |    |    |    |   |   |   |   |   |   |
| .25  | -2   |   |      |     |    |    |    |   |   |   |   |   |   |
| .5   | -1   |   |      |     |    |    |    |   |   |   |   |   |   |
| 1  | 0  |   |      |     |    |    |    |   |   |   |   |   |   |
| 2  | 1  |   |      |     |    |    |    |   |   |   |   |   |   |
| 4  | 2  |   |      |     |    |    |    |   |   |   |   |   |   |

a) Find the requested function.

b) State the domain of function created in part a.

c) Describe the transformation compared with  $f(x)$

d) Graph the logarithmic functions

8)  $f(x + 1) + 3$

9)  $f(x - 3) - 2$

10) Write the expression as a single logarithm. Write your answer with only positive exponents.  
 $2\log_3 x + 4\log_3 y - 5\log_3 z$

*Steps:*

- i) Make any coefficient an exponent of the logarithm it is in front of.*
- ii) Write a single log with the base that is common.*

*iii) Place the arguments in the correct place, with the correct signs between them.*

*There should not be any plus or minus signs used on this step.*

*If there is no subtraction, then there is no fraction.*

*If there is a minus sign, any argument whose logarithm is preceded by a minus sign belongs in the denominator of a fraction. The rest of the arguments belong in the numerator.*

11) Expand into sums and differences of logarithms (express exponents as multiplication).  $\log_3 \frac{x^2 y}{w^4 z}$

*Steps:*

*i) Write one logarithm with the appropriate base for each letter (variable). In this case we will have 4 logarithms as there are 4 variables.*

*ii) Put the appropriate sign between the logarithms. You may only use plus or minus signs on this step. If you use a times or divide sign you have made a mistake.*

*Put a minus sign in front of any logarithm whose variable is in the denominator, the rest get plus signs in front, or no sign in front if it is the first logarithm in your answer.*

*iii) Make each exponent a coefficient.*

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Chapter 6 Practice Test Part 2

#12-13: Solve the exponential equations, round your answer to 2 decimals.

12)  $3^x = 18$

13)  $3^x = 2^{x-1}$

#14-19: Solve the logarithmic equations, round to 2 decimals when needed.

14)  $\log_3 x = 4$

15)  $\ln x = 2$

16)  $\log_2(x+1) = 5$

17)  $\ln(4x-8) = \ln(3x-1)$

18)  $\log_2(x+2) - \log_2(x-2) = 1$

*Step 1: Use the minus to divide rule to write the left side with one logarithm.*

*Step 2: Scratch out the log and switch the  $\frac{x+2}{x-2}$  and the 1.*

*Step 3: Make each side a fraction and solve by cross multiplying.*

19)  $\log_2(x+2) + \log_2(x-2) = 5$

(be sure to check for extraneous solutions)

*Step 1: Use the plus to times rule to write the left side with one logarithm.*

*Step 2: Foil*

*Step 3: Scratch out the log and switch the  $x^2 - 4$  and the 5*

*Step 4: Set equal to zero and solve by factoring*

*Step 5: Check your answer. The positive answer will check and the negative will not check.*

*Step 6: Write the answer that checks as your solution.*

20) How long will it take an initial investment of \$10,000 to double if it is expected to earn 5% interest compounded continuously? (Round to 1 decimal place) (use formula  $A = Pe^{rt}$ ).

*Hint: A = future value P = starting value r = interest rate as a decimal t = number of years*

21) The population of a city is expected to double in 20 years. The city currently has 5,000 residents. How long will it take to get to 30,000 residents? (Round final answer to 2 decimals.)  
(use formula  $P(t) = P_0e^{kt}$ , round  $k$  to 3 decimals)

$P(t)$  = future population                       $P_0$  = current population

$k$  = growth rate (will need to solve for this using the information in the first 2 sentences)

$t$  = time in years

**Step one:** Use this part of the problem to solve for  $k$ .

*The population of a city is expected to double in 20 years. The city currently has 5,000 residents.*

**Step two:** use the value of  $k$  found in step 1, along with this part of the problem to solve for  $t$ .

*The city currently has 5,000 residents. How long will it take to get to 30,000 residents? (Round final answer to 2 decimals.)*