

Section 1.8: Operations with Fractions, Decimals and Percent
 Chapter 1: Introduction to Algebra

Multiply decimals

- Step 1. Determine the sign of the product.
- Step 2. Write in vertical format, lining up the numbers on the right. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
- Step 3. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors.
- Step 4. Write the product with the appropriate sign.

For Example:

$$3.77 \times 2.8 = ?$$

$$\begin{array}{r}
 3.77 \text{ (2 decimal places)} \\
 \times 2.8 \text{ (1 decimal place)} \\
 \hline
 3016 \\
 +754 \\
 \hline
 10.556 \text{ (3 decimal places)}
 \end{array}$$

Example Find the product of 8.34 and (-4.2) .

Step 1
 Multiply the numbers as if they were whole numbers.

$$\begin{array}{r}
 8.34 \\
 \times 4.2 \\
 \hline
 1668 \\
 3336 \\
 \hline
 35028
 \end{array}$$

You do not have to line up decimal points when multiplying.

Step 3 Count over 3 places in the product and write the decimal point. Count from *right to left*.

$$\begin{array}{r}
 8.34 \leftarrow 2 \text{ decimal places} \\
 \times 4.2 \leftarrow 1 \text{ decimal place} \\
 \hline
 1668 \quad 3 \text{ total decimal places} \\
 3336 \\
 \hline
 35.028 \leftarrow 3 \text{ decimal places in product}
 \end{array}$$

Step 4 The factors have *different* signs, the product is negative: $8.34 \times (-4.2) = -35.028$.

Multiplication by powers of 10

Multiply by positive powers of 10

$$8.265 \times 10^1 = 82.65$$



Shift decimal point **one** place to the **right**

$$8.265 \times 10^2 = 826.5$$



Shift decimal point **two** places to the **right**

Multiply by negative powers of 10

$$398.26 \times 10^{-1} = 39.826$$



Shift decimal point **one** place to the **left**

$$398.26 \times 10^{-2} = 3.9826$$



Shift decimal point **two** places to the **left**

Section 1.8: Operations with Fractions, Decimals and Percent
Chapter 1: Introduction to Algebra

#1 – 8: Multiply

1) 1.2×3.4

2) 4.2×5.7

3) -6.21×2.4

4) -3.14×2.3

5) 3.02556×10^3

6) 4.23587×10^4

7) 6.02×10^5

8) 5.1×10^3

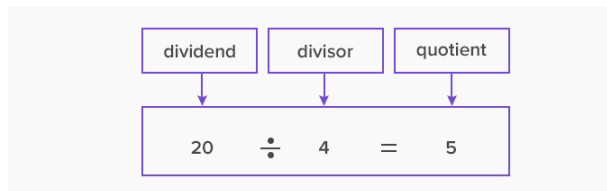
Section 1.8: Operations with Fractions, Decimals and Percent
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Divide decimals

- Step 1. Determine the sign of the quotient.
- Step 2. Make the divisor a whole number by “moving” the decimal point all the way to the right. “Move” the decimal point in the dividend the same number of places—adding zeros as needed.
- Step 3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
- Step 4. Write the quotient with the appropriate sign.

Remember these terms for division:

$$\begin{array}{r}
 6 \leftarrow \text{quotient} \\
 4 \overline{) 24} \leftarrow \text{dividend} \\
 \uparrow \\
 \text{divisor}
 \end{array}$$



For Example:

Find the quotient.

$$55.318 \div 3.4 \rightarrow 3.4 \overline{) 55.318} \quad \text{Write in standard form.}$$

$$\begin{array}{r}
 3.4 \overline{) 55.318} \\
 \hline
 \end{array}$$

Move decimal point in divisor and dividend.

$$\begin{array}{r}
 16.27 \\
 3.4 \overline{) 55.318} \\
 \underline{-34} \\
 213 \\
 \underline{-204} \\
 91 \\
 \underline{-68} \\
 238 \\
 \underline{-238} \\
 0
 \end{array}$$

Keep dividing until quotient repeats or comes out evenly.

Add zeros on right of dividend as needed.

The quotient is 16.27.

#9-12: Divide

9) $7.68 \div 3.2$

10) $17.22 \div 2.1$

11) $-21.73 \div 4.1$

12) $-13.33 \div 3.1$

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Chapter 1: Introduction to Algebra

A **percent** is a ratio whose denominator is 100. Percent means per hundred. We use the percent symbol, %, to show percent. Since a percent is a ratio, it can easily be expressed as a fraction.

Convert percent to decimal

Percent to Decimal Conversion

To Convert a Percentage into its Decimal Value:

- Remove the % sign
- Put a decimal point onto the end of the number
- Move this decimal point two places to the left
- Fill any empty places with a zero
- Remove any "trailing" zeroes off the very end of the answer
- Always have a Zero in front of the decimal point if the original % value in the question was less than 100 %

$$50\% = 50. = .50 = .5 = 0.5 \checkmark$$

Convert decimal to percent

Decimals to Percentages - EXAMPLES

To Convert a Decimal Value into a Percentage:

- Move the decimal point two places to the Right, and zero fill gaps
- Put a % sign onto the end of the answer

$$0.2 = 0.2__ = 020. = 020 = 20 = 20\% \checkmark$$

$$0.02 = 0.02__ = 002. = 002 \text{ (Remove Leading 0's)} = 2\% \checkmark$$

$$2.22 = 2.22__ = 222. = \text{(Remove Decimal Point)} = 222\% \checkmark$$

$$0.0002 = 0.0002__ = 000.02 = 0.02 = 0.02\% \checkmark$$

Convert decimal to a fraction

Decimal → Fraction

$$.15 = \frac{15}{100} \stackrel{\div 5}{=} \frac{3}{20}$$

The last digit is in the hundredths place.

Use the place value of the last digit to write as fraction with denominator of 10, 100, 1000 etc. Then simplify the fraction if possible.

Convert fraction to a decimal

- Convert the fraction to a long division problem and perform the division.

For Example: Convert $\frac{1}{4}$ to a decimal.

Just set up the long division and perform the division.

$$\begin{array}{r} 0.25 \\ 4 \overline{) 1.00} \\ \underline{- 8} \\ 20 \\ \underline{- 20} \\ 0 \end{array}$$

denominator numerator

Answer: $\frac{1}{4} = 0.25$

Section 1.8: Operations with Fractions, Decimals and Percent
Chapter 1: Introduction to Algebra

Convert fraction to percent

- First perform long division to change the fraction to a decimal
- Second move the decimal two to the right to create a percent

For Example: Convert $\frac{1}{4}$ to a percent.

First:

$$\begin{array}{r} 0.25 \\ 4 \overline{) 1.00} \\ \underline{- 8} \\ 20 \\ \underline{- 20} \\ 0 \end{array}$$

denominator ← 4 ← numerator

Thus $\frac{1}{4} = 0.25$

Second move the decimal two places to the right to create a percent.

$$\frac{1}{4} = 0.25 = 25\%$$

Convert fraction to percent – alternative method

Fraction to Percentage Conversion

To Convert any Fraction into a Percentage:

- Make an Improper Fraction for Mixed Numbers
- Multiply the Fraction by $\frac{100}{1}$ then add a % sign
- Simplify our result by Reducing Down if needed

$$\frac{1}{5} = \frac{1}{5} \times \frac{100}{1} = \frac{100}{5} \text{ (Divide by 5)} = \frac{20}{1} = 20\% \checkmark$$

$$2\frac{3}{4} = \frac{(4 \times 2 + 3)}{4} \times \frac{100}{1} = \frac{11}{4} \times \frac{100}{1} = \frac{1100}{4} = 275\% \checkmark$$

Section 1.8: Operations with Fractions, Decimals and Percent
Chapter 1: Introduction to Algebra

Convert percent to fraction

- First convert the percent to a decimal by moving the decimal two places to the right
- Second create a fraction with the appropriate power of 10 in the denominator
- Third reduce the fraction

For Example: Convert 0.345% to a fraction

First: convert 0.345% to a decimal by moving the decimal two places to the right.

$$0.345\% = 34.5$$

Second: right as a fraction. The denominator will be 10 as there is only one number after the decimal point.

$$0.345\% = 34.5 = \frac{345}{10}$$

Third reduce the fraction:

$$0.345\% = 34.5 = \frac{345}{10} = \frac{5 \times 69}{5 \times 2} = \frac{69}{2}$$

Answer: $0.345\% = \frac{69}{2}$

Principal Square Roots

Principal square root of m (\sqrt{m}) is the **positive number** which when squared equals m .

- If n is a positive number and $n^2 = m$ then $\sqrt{m} = n$
- NOTE $\sqrt{\quad}$ the principal square root symbol requires a positive answer

Principal Square Root Notation:

\sqrt{m} is read "the principle square root of m ." 

For example: $\sqrt{25} = 5$ (since 5 is a positive number and $5^2 = 25$)

13) Convert 30.251% to a decimal.

14) Convert 24.155 % to a decimal.

15) Convert 25.4% to a fraction.

16) Convert 12.3% to a fraction.

17) Write 0.12 as a fraction, reduced to lowest terms.

18) Write 0.24 as a fraction, reduced to lowest terms.

19) Write 6.123 as a percent.

20) Write 4.0225 as a percent.

21) Convert $\frac{4}{5}$ to a decimal.

22) Convert $\frac{2}{5}$ to a decimal.

23) Convert $\frac{6}{24}$ to a percent.

24) Convert $\frac{9}{12}$ to a percent.

#25 – 26: Complete the table

25)			26)		
Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{2}$			$\frac{3}{4}$		
	0.8			0.28	
		75%			60%

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Numbers can be classified into 5 broad categories. The categories are as follows: (Note that some numbers count in more than one category.)

Counting Numbers (Natural Numbers)	1, 2, 3, 4, ...
Whole Numbers	0, 1, 2, 3, 4...
Integers	..., -3, -2, -1, 0, 1, 2, 3...
Rational Number	Examples: $5, \frac{7}{8}, \frac{1}{3}, 0.97, \overline{0.21}, \sqrt{16}$
Irrational Number	Examples: $\pi, e, \sqrt{8}, 0.30312300003...$

Counting numbers (also called natural numbers) are those used to count physical objects in the real world, such as 1, 2, 3, 4 ...

- 6 is a counting number as it can be used to count many physical objects.

Whole numbers are counting numbers with the number 0 added.

- 9 is a whole number as it is a counting number
- 0 is a whole number as 0 is included as a whole number

Integers are all counting numbers, the opposite of every counting number and 0.

- -3 is an integer as it is the opposite of the counting number 3

Rational numbers are numbers that can be written as a ratio of two integers.

Its decimal form does stop or repeat.

NOTE: Any number with a repeating decimal can be written as the ratio of two integers. Thus, any number with a repeating decimal is a rational number.

- $\frac{2}{3}$ is a rational number as it is a fraction with integers in the numerator and the denominator.
- $42.5\overline{34}$ is a rational number as it has a repeating decimal. With a bit of work, $42.5\overline{34}$ can be written as the fraction $\frac{42109}{990}$.
- -7 is a rational number as it can be written as $\frac{-7}{1}$

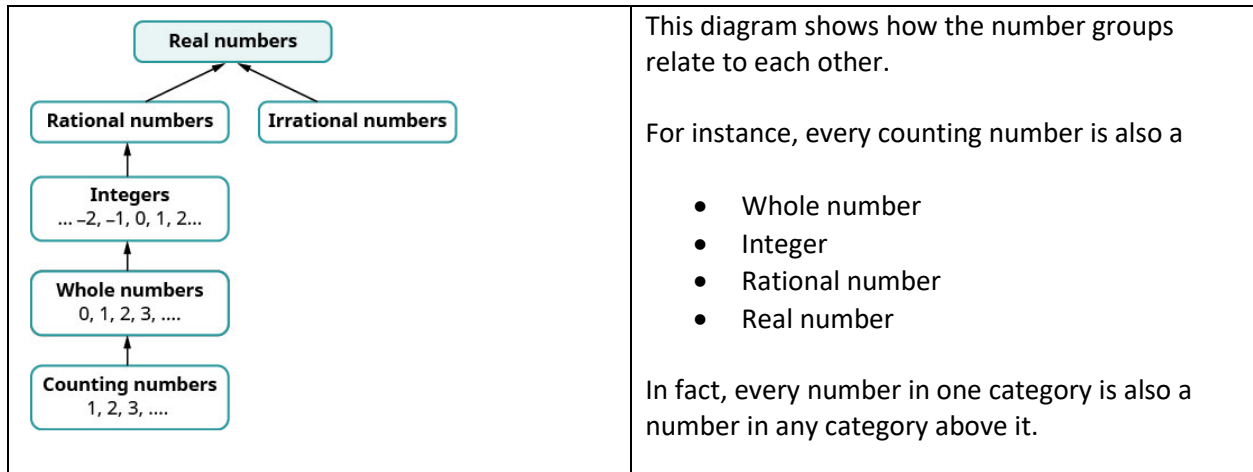
An **irrational number** is a number that cannot be written as the ratio of two integers. Its decimal form does not stop and does not repeat.

- 15.435678234... is an irrational number, assuming the decimal does not terminate nor repeat.
- Any number that contains π (3.14 ...) or e (2.718..) are irrational as the decimal does not terminate nor repeat.
- $\sqrt{5}$ is an irrational number. Its decimal form does not stop, nor repeat. In fact, the square root of any number that is not a perfect square is an irrational number.

Real number is any number that is either rational or irrational.

- All numbers listed as examples above are real numbers.

Section 1.8: Operations with Fractions, Decimals and Percent
Chapter 1: Introduction to Algebra



27) Which of the numbers are counting numbers?

$7, \pi, \frac{5}{3}, \sqrt{16}, \sqrt{7}, -12, 5\frac{1}{3}, 0.\overline{23}, 6.251, 0, 14, -\frac{5}{4}$

28) Which of the numbers are counting numbers?

$2\pi, \frac{-11}{7}, 2.3\overline{56}, -24, 0, 4.221, \sqrt{4}, \sqrt{10}$

29) Which of the following are integers?

$7, \pi, \frac{5}{3}, \sqrt{16}, \sqrt{7}, -12, 5\frac{1}{3}, 0.\overline{23}, 6.251, 0, 14, -\frac{5}{4}$

30) Which of the following are integers?

$2\pi, \frac{-11}{7}, 2.3\overline{56}, -24, 0, 4.221, \sqrt{4}, \sqrt{10}$

31) Which of the numbers are rational numbers?

$7, \pi, \frac{5}{3}, \sqrt{16}, \sqrt{7}, -12, 5\frac{1}{3}, 0.\overline{23}, 6.251, 0, 14, -\frac{5}{4}$

32) Which of the numbers are rational numbers?

$2\pi, \frac{-11}{7}, 2.3\overline{56}, -24, 0, 4.221, \sqrt{4}, \sqrt{10}$

33) Which of the following are irrational numbers?

$7, \pi, \frac{5}{3}, \sqrt{16}, \sqrt{7}, -12, 5\frac{1}{3}, 0.\overline{23}, 6.251, 0, 14, -\frac{5}{4}$

34) Which of the following are irrational numbers?

$2\pi, \frac{-11}{7}, 2.3\overline{56}, -24, 0, 4.221, \sqrt{4}, \sqrt{10}$