

Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

Determine if a graph represents y as a function of x .

We learned how to determine if a relation represents y as a function of x in the previous section.

1) y is NOT a function of x if there are two or more points with the same x – *value*, but *different* y – *values*.

2) y is a function of x if each x has a different y .

To determine if a graph represents y as a function of x can be difficult using what we know so far. This is because points on a graph are not always marked. This can make it hard to find points that have the same x – *values* with different y – *values*.

There is technique called the vertical line test that is often used to determine if a graph represents y as a function of x .

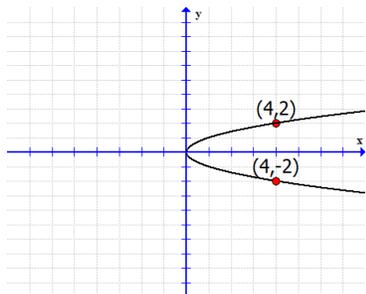
The **vertical line test** is a quick way to determine if a graph represents y as a function of x without the need of listing different points with the same x – *values* but different y – *values*.

Vertical line test:

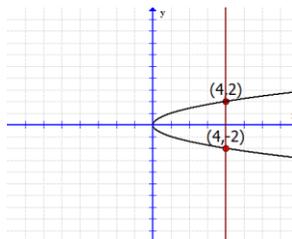
- If a vertical line can be drawn to touch the graph of a function in more than one place, then y is NOT a function of x .
- If it is not possible to draw a vertical line to touch the graph of a function in more than one place, then y is a function of x .

For Example:

Use the vertical line test to determine if the graph depicts y is a function of x .



Solution: **y is NOT a function of x** (as it is possible to draw a vertical line to touch the graph in more than one place.)

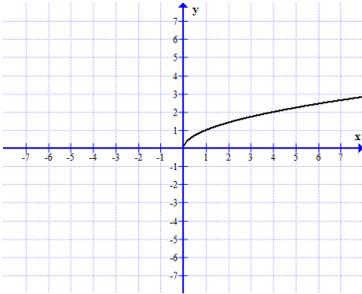


Notice, the vertical line test has found two points on the graph $\{ (4,2) \text{ and } (4,-2) \}$ that have the same x with different y 's.

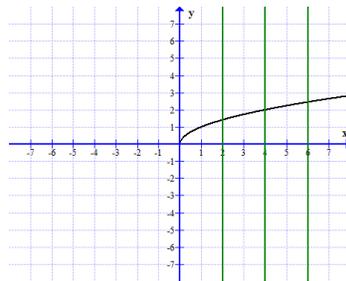
Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

For Example:

Use the vertical line test to determine if the graph depicts y is a function of x .



Solution: **y is a function of x** (as no vertical line can be drawn to touch the graph in more than one place.)



Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

Find the Domain and Range from the graph of a
continuous function using INTERVAL NOTATION

Interval notation has two types of symbols:

<p>1) Parenthesis () Use round parenthesis when: a) point is marked with an open circle \circ b) value is infinity ∞</p>	<p>2) Bracket [] Use bracket when: a) point is marked with a closed circle b) point is an unmarked point on a graph</p>
---	---

Writing the domain and or range may require a bracket on one side of the interval and a parenthesis on the other.

- How to find **domain** from a graph of a continuous function and write answer in interval notation.

Step 1) Identify the x-coordinate of left-most the point on the graph. Place it after the appropriate symbol (or [.

Step 2) Identify the x-coordinate of the right-most point on the graph. Place it before the appropriate symbol) or].

- How to find **range** from a graph of a continuous function and write answer in interval notation.

Step 1) Identify the y-coordinate of lowest point on the graph. Place it after the appropriate symbol (or [.

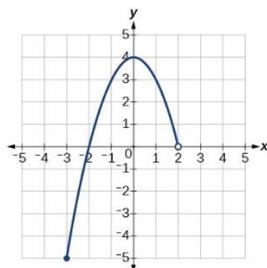
Step 2) Identify the y-coordinate of the highest point on the graph. Place it before the appropriate symbol) or].

It is common for a graph continue beyond the portion of the graph we can see; the domain and range may be greater than the visible portion of the graph.

We generally need ∞ for one or both sides of the interval of the domain and or range when a graph extends beyond what we can see.

Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

Find the domain and range of the graph below, write answer in interval notation.



Domain:

First: Find the x-coordinate of the point that is furthest left and decide whether to put a (or [before the number.

The point that is furthest left is (-3, -5). The x-coordinate of the point is $x = -3$.

“[” is needed as the point is marked with a closed circle.

The domain will start with [-3,

Second: Find the x-coordinate of the point that is furthest right and decide whether to put a) or] after it.

The point that is furthest right is (2,0). The x-coordinate is $x = 2$.

”)” is needed as the point is marked with an open circle.

The domain will end with ,2)

Answer: Domain [-3,2)

Range:

First: Find the y-coordinate of the bottom point and decide whether to put a (or [before the number.

The bottom point is (-3,-5). The y-coordinate of the top point is $y = -5$

“[” is needed as the point is marked with a closed circle.

The range will start with [-5

Second: Find the y-coordinate of the top point and decide whether to put a) or] after it.

The top point is (0,4). The y-coordinate of the top point is $y = 4$.

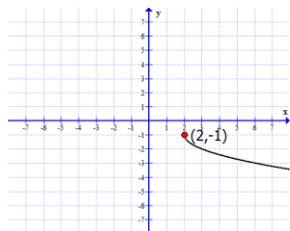
”)” will be needed to end the range as the point (0,4) is an unmarked point on the graph.

y to end the range: ,4]

Answer: Range [-5, 4]

Section 2.5: Domain and Range of graphs of functions
 Chapter 2: Functions, Linear equations, and inequalities

For Example: Find the domain and range of the graph below, write answer in interval notation.



Domain:

First: Find the x-coordinate of the point that is furthest left and decide whether to put a (or [before the number.

The point that is furthest left is (2, -1). The x-coordinate of the point is $x = 2$.

"[" is needed as the point is marked with a closed circle.

The domain will start with [2,

Second: Find the x-coordinate of the point that is furthest right and decide whether to put a) or] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-right edge of the x-axis. When a graph extends to the far-right edge of the x-axis, ∞ will be needed to end the domain.

"]" is needed: round parenthesis are always used for ∞ .

The domain will end with $, \infty)$

Answer: Domain $[-1, \infty)$

Range:

First: Find the y-coordinate of the bottom point and decide whether to put a (or [before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the bottom of the y-axis. When a graph extends to the bottom of the y-axis, $-\infty$ will be needed to start the range.

"(" is needed: round parenthesis are always used for ∞ and $-\infty$.

The range will start with $(-\infty,$

Second: Find the y-coordinate of the top point and decide whether to put a) or] after it.

The top point is (2, -1). The y-coordinate of the top point is $y = -1$.

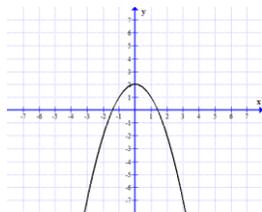
"]" will be needed to end the range as the point (2, -1) is marked with a closed circle.

y to end the range: $, -1]$

Answer: Range $(-\infty, -1]$

Section 2.5: Domain and Range of graphs of functions
 Chapter 2: Functions, Linear equations, and inequalities

Find the domain and range of the graph below, write answer in interval notation.



Domain:

First: Find the x-coordinate of the point that is furthest left and decide whether to put a) or] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-left edge of the x-axis. When a graph extends to the far-left edge of the x-axis, $-\infty$ will be needed to start the domain.

"(" is needed: round parenthesis are always used for ∞ .

The domain will start with $(-\infty$

Second: Find the x-coordinate of the point that is furthest right and decide whether to put a) or] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-right edge of the x-axis. When a graph extends to the far-right edge of the x-axis, ∞ will be needed to end the domain.

)" is needed: round parenthesis are always used for ∞ .

The domain will end with $), \infty)$

Answer: Domain $(-\infty, \infty)$

Range:

First: Find the y-coordinate of the bottom point and decide whether to put a (or [before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the bottom edge of the y-axis on both sides of the graph. When a graph extends to the bottom of the y-axis, $-\infty$ will be needed to start the range.

"(" is needed: round parenthesis are always used for ∞ and $-\infty$.

The range will start with $(-\infty,$

Second: Find the y-coordinate of the top point and decide whether to put a) or] after it.

The top point is (0,2). The y-coordinate of the top point is $y = 2$.

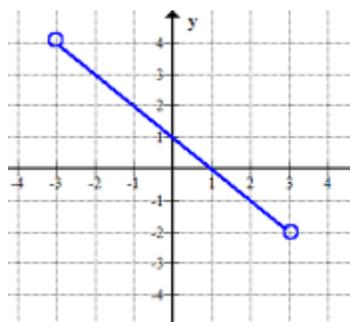
)" will be needed to end the range as the point (0,2) is unmarked.

y to end the range: $), 2]$

Answer: Range $(-\infty, 2]$

Section 2.5: Domain and Range of graphs of functions
 Chapter 2: Functions, Linear equations, and inequalities

Find the domain and range of the graph below, write answer in interval notation.



Domain:

First: Find the x – *coordinate* of the point that is furthest left and decide whether to put a (or [before the number.

The point that is furthest left is $(-3, 4)$. The x -coordinate of the point is $x = -3$.

“(“ is needed as the point is marked with an open circle.

The domain will start with $(-3,$

Second: Find the x – *coordinate* of the point that is furthest right and decide whether to put a) or] after it.

The point that is furthest right is $(3, -2)$.
 The x – *coordinate* is $x = 3$

The domain will end with $, 3)$

”)” is needed as the point is marked with an open circle.

Answer: Domain $(-3, 3)$

Range:

First: Find the y – *coordinate* of the bottom point and decide whether to put a (or [before the number.

The bottom point is $(3, -2)$. The y – *coordinate* of the bottom point is $y = -2$

“(“ is needed as the point is marked with an open circle.

The range will start with $(-2$

Second: Find the y – *coordinate* of the top point and decide whether to put a) or] after it.

The top point is $(-3, 4)$. The y – *coordinate* of the top point is $y = 4$.

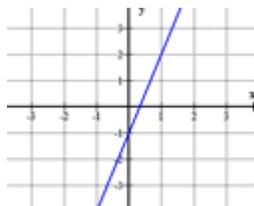
”)” will be needed to end the range as the point $(-3, 4)$ is marked with an open circle.

y to end the range: $, 4)$

Answer: Range $(-2, 4)$

Section 2.5: Domain and Range of graphs of functions
 Chapter 2: Functions, Linear equations, and inequalities

Find the domain and range of the graph below, write answer in interval notation.



Domain:

First: Find the x-coordinate of the point that is furthest left and decide whether to put a (or] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-left edge of the x-axis. When a graph extends to the far-left edge of the x-axis, $-\infty$ will be needed to start the domain.

"(" is needed: round parenthesis are always used for ∞ .

The domain will start with $(-\infty$

Second: Find the x-coordinate of the point that is furthest right and decide whether to put a) or] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-right edge of the x-axis. When a graph extends to the far-right edge of the x-axis, ∞ will be needed to end the domain.

)" is needed: round parenthesis are always used for ∞ .

The domain will end with $, \infty)$

Answer: Domain $(-\infty, \infty)$

Range:

First: Find the y-coordinate of the bottom point and decide whether to put a (or [before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the bottom edge of the y-axis on both sides of the graph. When a graph extends to the bottom of the y-axis, $-\infty$ will be needed to start the range.

"(" is needed: round parenthesis are always used for ∞ and $-\infty$.

The range will start with $(-\infty,$

Second: Find the y-coordinate of the top point and decide whether to put a (or [before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the top edge of the y-axis on both sides of the graph. When a graph extends to the bottom of the y-axis, ∞ will be needed to end the range.

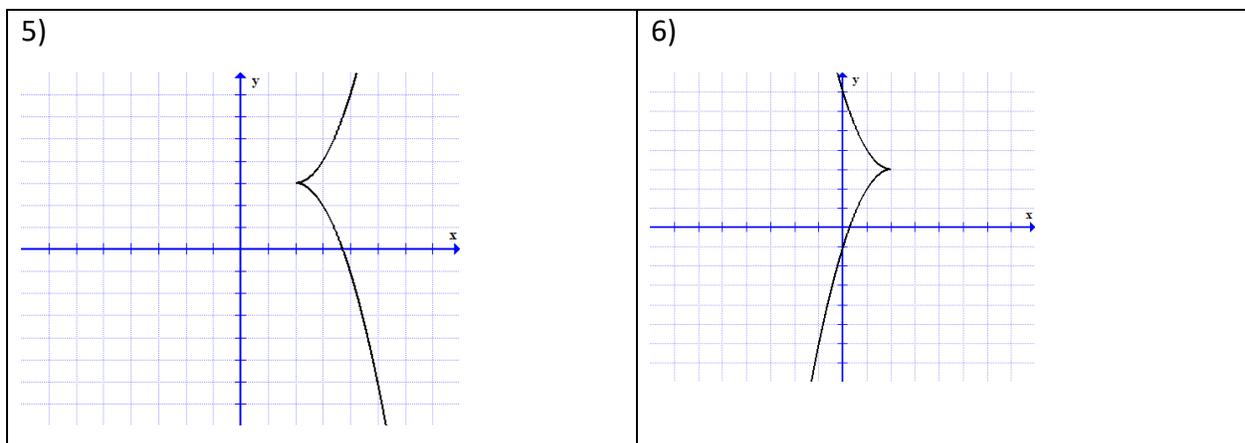
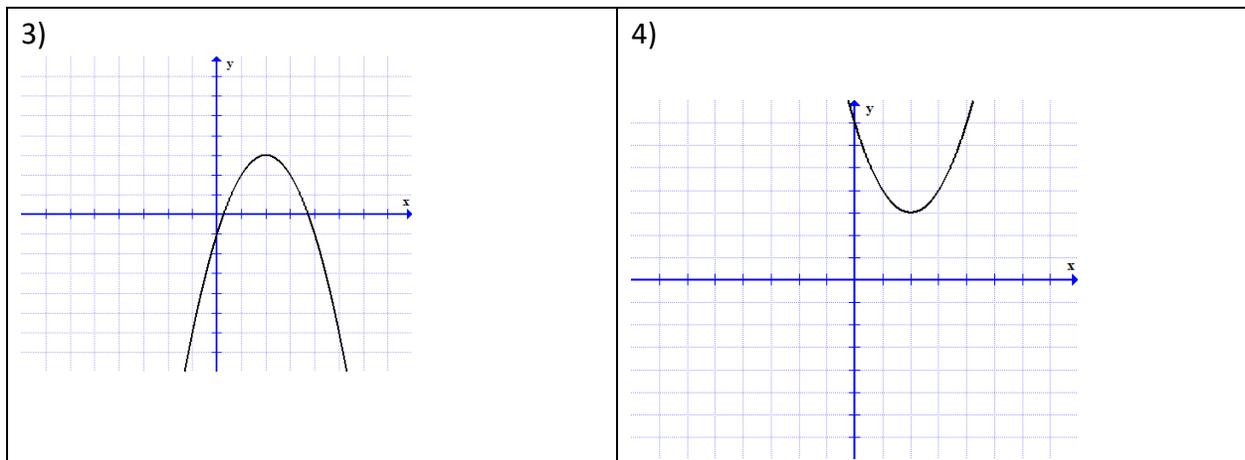
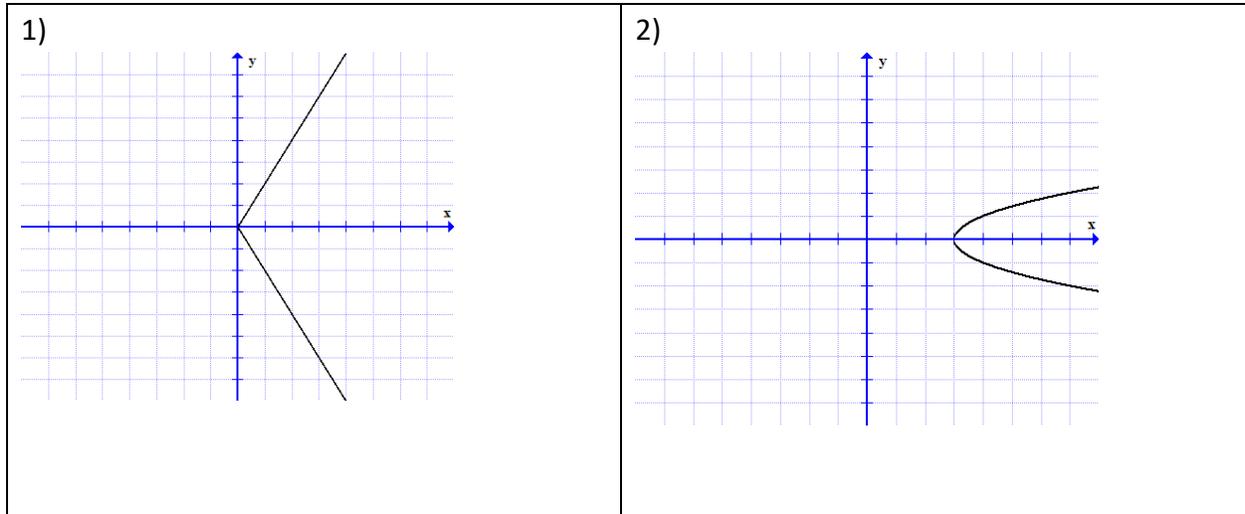
"(" is needed: round parenthesis are always used for ∞ and $-\infty$.

The range will end with $, \infty)$

Answer: Range $(-\infty, \infty)$

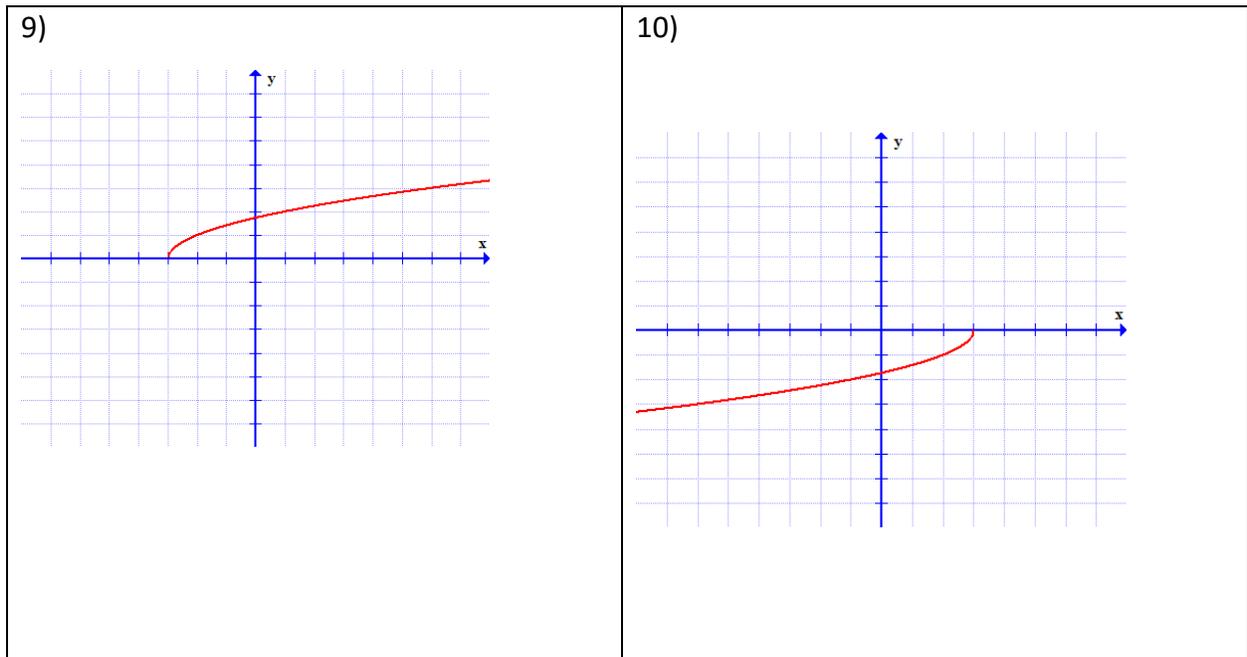
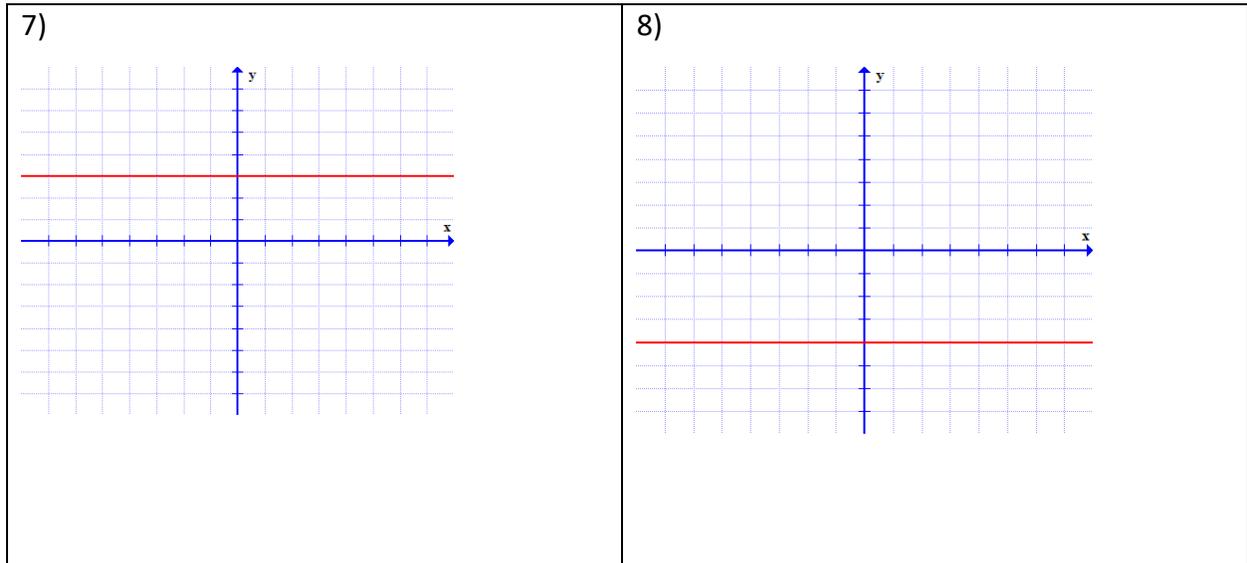
Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

#1-10: Use the vertical line test to determine if the graph represents y as a function of x .



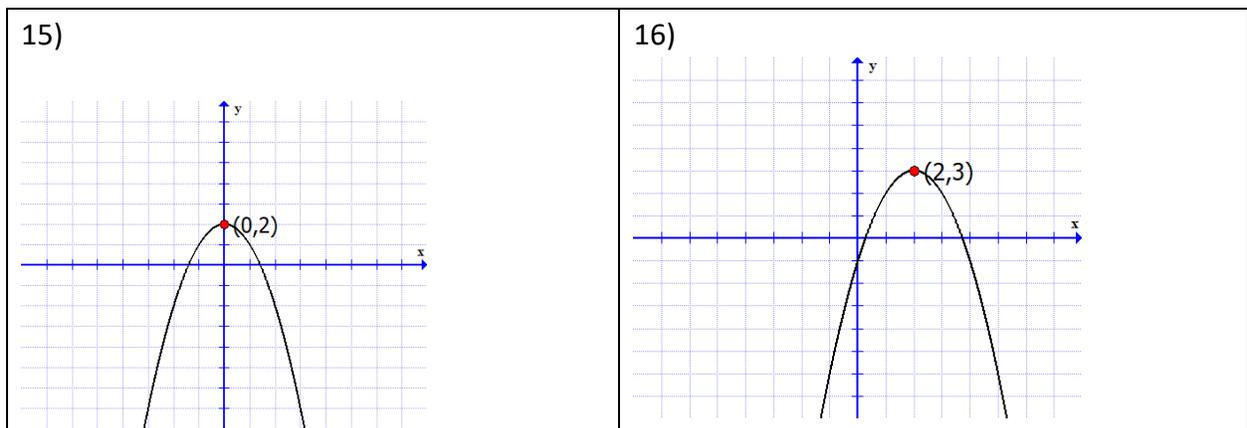
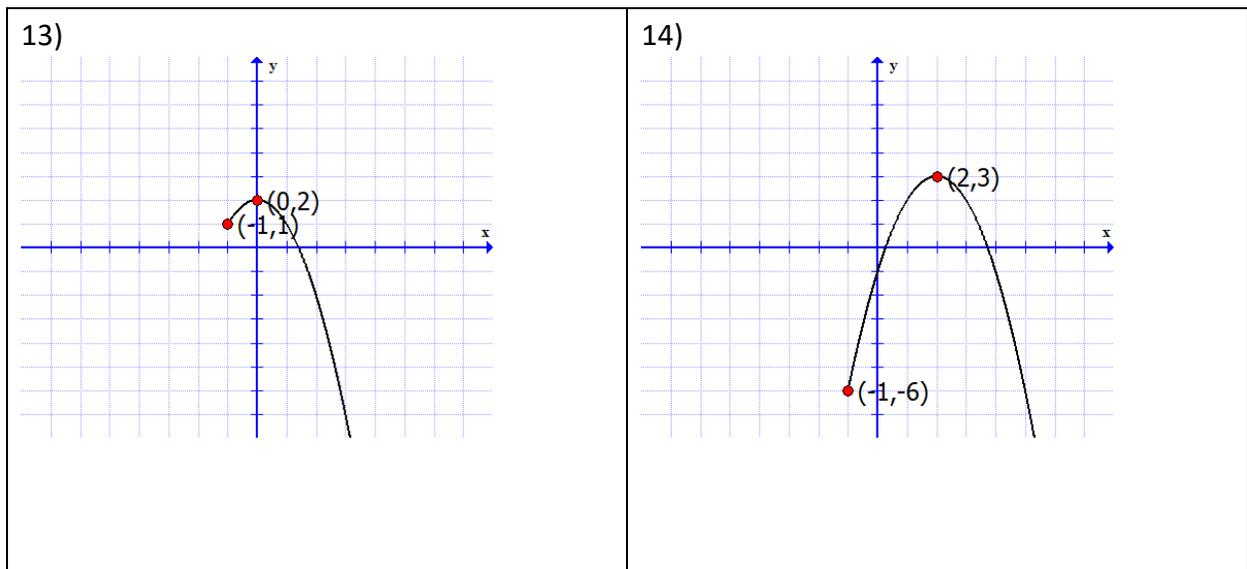
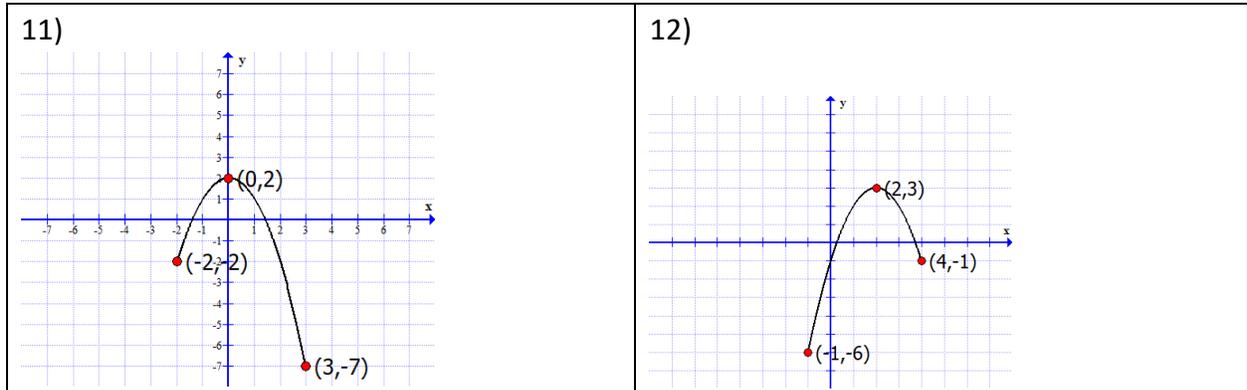
Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

#1-10 Continued: Use the vertical line test to determine if the graph represents y as a function of x .



Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

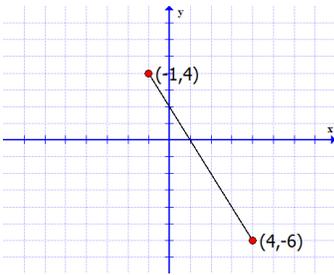
#11 – 30: Find the domain and range of function. Write your answer in interval notation.



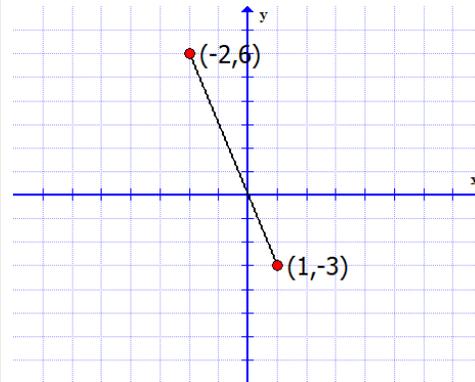
Section 2.5: Domain and Range of graphs of functions
Chapter 2: Functions, Linear equations, and inequalities

#11 – 30 Continued: Find the domain and range of function. Write your answer in interval notation.

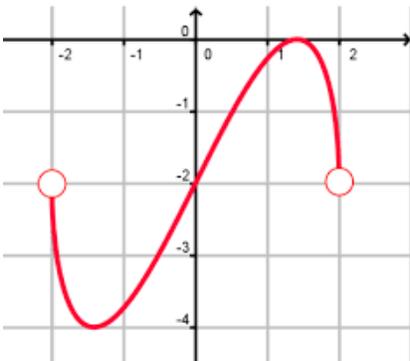
17)



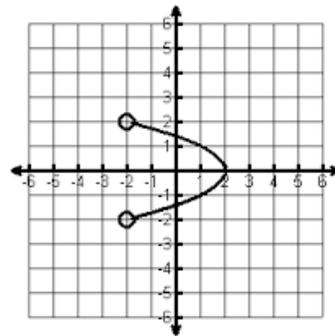
18)



19)

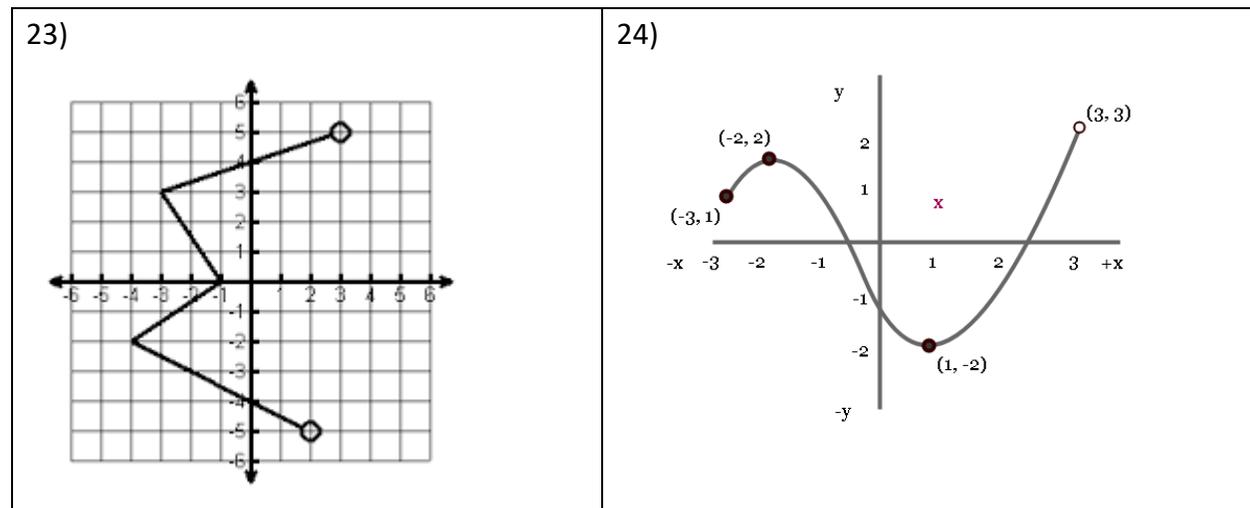
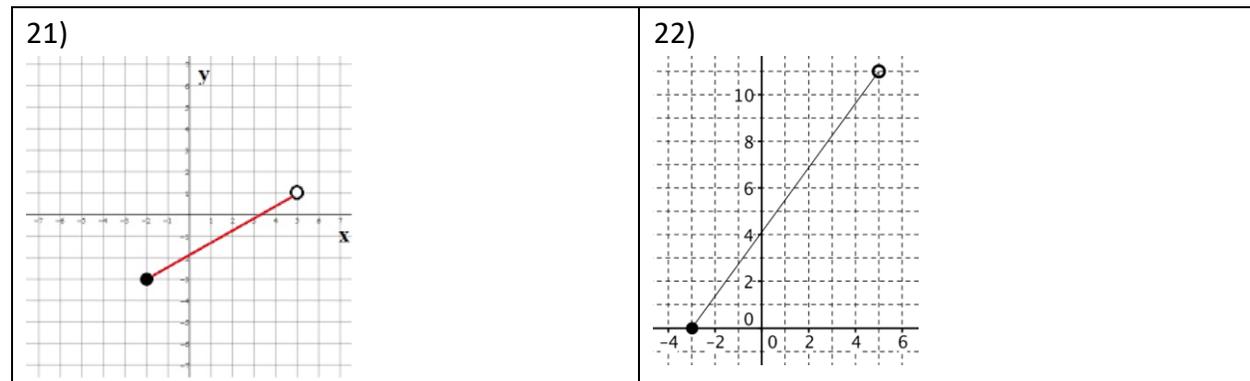


20)



Section 2.5: Domain and Range of graphs of functions
 Chapter 2: Functions, Linear equations, and inequalities

#11 – 30 Continued: Find the domain and range of function. Write your answer in interval notation.



Section 2.5: Domain and Range of graphs of functions
 Chapter 2: Functions, Linear equations, and inequalities

#11 – 30 Continued: Find the domain and range of function. Write your answer in interval notation.

