Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

same sign — opposite sign — always +

After any GCF is factored out, an expression is considered a difference of cubes provided:

- Only has 2 terms
- Minus sign between the terms
- Variables have exponents that are multiples of 3, such as x^3, y^6, z^9, w^{12} …
- Numbers are perfect cubes, such as $1, 8, 27, 64, 125, 216, 343, 512, \ldots$

If all 4 above are true after any GCF is factored out, then we factor as follows:

Step 1: Factor out GCF, if applicable
Step 2: Use the rule: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Step 3: Determine "a".

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable)

Step 4: Determine "b"

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable

Step 5: Place the numbers in the formula and simplify to get the answer.

Step 6: Check

Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

same sign — opposite sign — always +

After any GCF is factored out, an expression is considered a sum of cubes provided:

- Only has 2 terms
- Plus sign between the terms
- Variables have exponents that are multiples of 3, such as x^3, y^6, z^9, w^{12} …
- Numbers are perfect cubes, such as $1, 8, 27, 64, 125, 216, 343, 512$, …

If all 4 above are true after any GCF is factored out, then we factor as follows:

Step 1: Factor out GCF, if applicable
Step 2: Create template: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Step 3: Determine "a".

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable)

Step 4: Determine "b"

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable

Step 5: Place the numbers in the formula and simplify to get the answer.

Step 6: Check

For Example: factor $27x^3 - 64$

Step 1: Factor out GCF, if applicable

Step 2: Create Template:

$a^3 - b^3$

$= (a-b)(a^2 + ab + b^2)$

Step 3: Determine "a".

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable)

Step 4: Determine "b"

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable

Step 5: Place the numbers in the formula and simplify to get the answer.

Step 6: Check

Solution: There is no GCF.

Check to see if it is a difference of cubes:

- Only has 2 terms ✓
- Minus sign between the terms ✓
- Variables have exponents that are multiples of 3, $x^3, exponent\ of\ 3\ is\ a\ multiple\ of\ 3$ ✓
- Numbers are perfect cubes, $27 = 3^3, 64 = 4^3$ ✓

Problem is a difference of cubes.

Step 1: There is no GCF, skip to step 2.

Step 2: Create Template: $(a-b)(a^2 + ab + b^2)$

Step 3: Determine "a". $a = \sqrt[3]{27}x^{3/3} = 3x$

$a = 3x$

Step 3: Determine "b". $b = \sqrt[3]{64} = 4$

b = 4

Step 5: Place the numbers in the formula and simplify to get the answer.

$27x^3 - 64 = (3x-4)((3x)^2 + (3x)(4) + (4)^2)$

= Answer: $(3x-4)(9x^2 + 12x + 16)$

Step 6: Check

$9x^2 + 12x + 16$

$\times \quad 3x - 4$

$-36x^2 - 48x - 64$

$27x^3 + 36x^2 + 48x$

$27x^3 - 64$ ✓

For Example: Factor $125x^3 + y^6$

Step 1: Factor out GCF, if applicable

Step 2: Create Template:

$a^3 + b^3$

$= (a + b)(a^2 - ab + b^2)$

Step 3: Determine "a".

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable)

Step 4: Determine "b"

- Cube root any numbers in the first term. (if applicable)
- Divide the exponent of the first term's variable by 3. (if applicable

Step 5: Place the numbers in the formula and simplify to get the answer.

Step 6: Check

Solution: There is no GCF.

Check to see if it is a sum of cubes:

- Only has 2 terms ✓
- Plus sign between the terms ✓
- Variables have exponents that are multiples of 3, $x^3, exponent\ of\ 3\ is\ a\ multiple\ of\ 3$ y^6 exponent of 6 is a multiple of 3. ✓
- Numbers are perfect cubes, $125 = 5^3$, ✓

Problem is a sum of cubes.

Step 1: There is no GCF, skip to step 2.

Step 2: Create Template: $(a + b)(a^2 - ab + b^2)$

Step 3: Determine "a". $a = \sqrt[3]{125}x^{3/3} = 5x$

$a = 3x$

Step 3: Determine "b". $b = y^{6/3} = y^2$

b = y^2

Step 5: Place the numbers in the formula and simplify to get the answer.

$125x^3 + y^6$

$= (5x + y^2)((5x)^2 - (5x)(y^2) + (y^2)^2)$

= Answer: $(5x + y^2)(25x^2 - 5xy^2 + y^4)$

Step 6: Check

$25x^2 - 5xy^2 + y^4$

$\times \quad 5x + y^2$

$25x^2y^2 - 5xy^4 + y^6$

$125x^3 - 25x^2y^2 + 5xy^4$

$125x^3 + y^6$ ✓

Section 5.6: Factoring Sums and Differences of Cubes Chapter 5: Factoring

#1-42: Completely factor the binomials, remember to factor out the GCF first when applicable (if a problem is prime say so).

1) $x^3 + 8$

2) $y^3 + 64$

3) $x^3 - 8$

4) $y^3 - 64$

5) $b^3 + 27$

6) $x^3 + 125$

7) $b^3 - 27$

8) $x^3 - 125$

9) $x^3 + 64$

10) $y^3 + 27$

11) $x^3 - 64$

12) $y^3 - 27$

13) $8x^3 - 27$

14) $125x^3 - 64$

15) $8x^3 + 27$

16) $125x^3 + 64$

17) $27x^3 - 125$

18) $64x^3 - 27$

19) $64x^3 - y^3$

20) $27x^3 - y^3$

21) $x^9 - y^3$

22) $y^6 - x^3$

23) $2x^6 - 54$

24) $3x^6 - 81$

25) $125x^9 - y^6$

26) $8y^9 - 125x^6$

27) $16x^3 - 54$

28) $3x^3 - 81$

29) $3x^3 + 24$

30) $2x^3 + 128$

31) $x^4 - 8x$

32) $y^4 - 64y$

33) $6x^4 - 48x$

34) $3y^4 - 192y$

35) $8x^5 + 125x^2$

36) $3x^5 - 375x^2$

37) $-x^3 + 27$

38) $-y^3 + 64$

39) $64x^3 + 27$

40) $8x^3 + 125$

41) $x^6 + 8$

42) $x^9 + 64$