

Section 1.2: Introduction to sets - Continued.

There are three ways to define a set:

- 1) Word description
- 2) Set-builder form
- 3) Roster form

Word Description: Simply using words to describe the elements of a set. There may be more than one correct way to define a set using a word description.

It will be helpful if we remember the definition of Natural numbers when we complete the next few examples.

The set of Natural numbers is a set you should be familiar with. The set of Natural numbers is often assigned the letter N , and it contains integers greater than 0.

$N = \text{set of Natural numbers} = \{1,2,3,4,5,\dots\}$

Example: Write a word description of the set: $A = \{1,2,3,4,5\}$

Answer 1: A is the set of Natural numbers between 1 and 5 inclusive.

Answer 2: A is the set of Natural numbers less than 6.

Both word descriptions are equal to the set $A = \{1,2,3,4,5\}$

There are many correct answers. When asked for a word description we only need to give one of many possible correct answers.

Example: Write a word description of the set: $A = \{4,5,6,7,\dots\}$

Answer 1: A is the set of Natural numbers greater than or equal to 4.

Answer 2: A is the set of Natural numbers greater than 3.

Both word descriptions produce the set $A = \{4,5,6,7,\dots\}$

Homework #1 – 8. Write a word description of each set

1) $D = \{2, 3, 4, \dots\}$

2) $B = \{4, 5, 6, \dots\}$

3) $A = \{1, 2, 3, 4, 5, 6, 7\}$

4) $E = \{5, 6, 7, 8, 9, 10\}$

5) $D = \{\text{Bashful, Doc, Dopey, Grumpy, Happy, Sleepy, Sneezy}\}$

6) $B = \{\text{California, Oregon, Washington}\}$

7) $V = \{a, e, i, o, u\}$

8) $T = \{1, 4, 9, 16, 25, 36, \dots\}$

Set-Builder Form: A set denoted in formal set builder notation has three parts contained inside set braces:

- 1) variable
- 2) vertical bar separator (or a colon)
- 3) logical predicate.

There is a variable written to the left of a vertical bar separator, and a rule (predicate) to the right of the vertical bar separator. These three parts are contained in curly brackets.

This is an example of a set defined using set-builder notation.

$$A = \{x \mid x \text{ is a natural number, } x < 3\}$$

Or equivalently

$$A = \{x \mid x \in \mathbb{N}, x < 3\}$$

- In this notation the curly braces are set braces.
- x is a variable, meaning it may take on a variety of values.
- The vertical bar stands for the phrase “with the property that.”
- A comma in this context means “and.”

I would read this as: Set A equals the set of all values for x with the property that x is a Natural number and x is less than 3.

That’s quite an involved description of set. I translated the symbols directly to get the description. There are better ways to describe set A .

Here is a better word description for set A : A is the set of Natural numbers that are less than 3.

It has the same meaning as the direct translation, but it is easier to understand.

Example: Write in set-builder notation: $A = \{2,3,4,5,\dots\}$

Start by creating the left side and the vertical bar that is required. I usually put an x before the vertical bar, unless the problem is worded in such a way that another letter makes sense.

$$A = \{x \mid$$

Next I need to figure out what to write after the vertical bar. The set A represents all natural numbers that are greater than or equal to 2. I need to represent all natural numbers greater than or equal to 2 as the "logical predicate". There are many correct ways to finish this problem off. Here are two correct answers. There are other correct answers.

$$A = \{x \mid x \text{ is a Natural number and } x \text{ is greater than } 1\}$$

Here is another correct answer:

$$A = \{x \mid x \text{ is a Natural number and } x \text{ is greater than or equal to } 2\}$$

Here is another correct answer:

$$A = \{x \mid x \in N, x \geq 2\}$$

Here is another correct answer:

$$A = \{x \mid x \in N, x > 1\}$$

There are many correct answers. Your answer is correct as long as it has the correct form, and the description yields the desired set.

Example: Write in set-builder notation: A = the set of natural numbers between 3 and 7 exclusive.

Start by creating the left side and the vertical bar that is required. I usually put an x before the vertical bar, unless the problem is worded in such a way that another letter makes sense.

$$A = \{x \mid$$

Next I need to figure out what to write after the vertical bar. I know A represents the numbers 4,5 and 6.

When x is between 2 numbers, I need to write x between the two numbers (imagine that), and there need to be less than or less than or equal signs between the numbers and the x . Both of these sets represent the numbers 4,5,6.

$A = \{x \mid x \in N, 3 < x < 7\}$ (x is between 3 and 7 and doesn't include the 3 or 7 because there is no "or equal")

$A = \{x \mid x \in N, 4 \leq x \leq 6\}$ (x is between 4 and 6 and it includes the 4 and 6 because of the equal)

Homework #9 – 16: Write the sets in set-builder form. (There are many correct answers.)

9) $D = \{2, 3, 4, \dots\}$

10) $A = \{3, 4, 5, 6, \dots\}$

11) $A = \{1, 2, 3, 4, 5, 6, 7\}$

12) $B = \{1, 2, 3, 4, 5, 6\}$

13) P = set of natural numbers between 3 and 11 inclusive

14) P = set of natural numbers between 4 and 9 inclusive

15) P = set of natural numbers between 3 and 11 exclusive

16) P = set of natural numbers between 4 and 9 exclusive

Roster, or List Form: Is a listing of all of the elements of the set using set braces.

Example: Write the set in roster form. A = the set of Natural numbers between 3 and 7 exclusive.

I need to figure out the elements of A , and then put them in set braces.

This set contains natural numbers between 3 and 7. It doesn't include the 3 nor the 7 because of the word exclusive.

Answer: $A = \{4,5,6\}$

Example: Write the set in roster form. A = the set of Natural numbers greater than 5.

This is an infinite set. I will need to use the ... in my answer.

5 won't be in my answer. The key word in the problem would need to be "greater than or equal to" for 5 to be part of my answer.

Answer: $A = \{6,7,8,\dots\}$

Example: Write the set in roster form. $D = \{m \mid m \in N, 4 \leq m < 8\}$

This represents all Natural numbers between 4 and 8. It includes the 4 because of the "or equal" after the 4, but doesn't include the 8 as the less than before the 8 doesn't have the "or equal" part.

Answer: $D = \{4,5,6,7\}$

Example: Write the set in roster form. $B = \{x \mid x + 1 = 9\}$

I need to solve $x+1 = 9$

I get 8.

Answer: $B = \{8\}$

Homework # 17 – 32. Write sets in roster form.

- 17) The set A of natural numbers between 5 and 10 inclusive.
- 18) The set A of even natural numbers between 5 and 10 inclusive.
- 19) The set A of even natural numbers greater than 4.
- 20) The set A of odd natural numbers greater than or equal to 5.
- 21) The set A of natural numbers greater than 4.
- 22) The set A of natural numbers greater than 5.
- 23) The set A of natural numbers that are negative.
- 24) The A set of natural numbers less than 0.
- 25) $B = \{ k \mid k+1=6 \}$
- 26) $B = \{ x \mid x - 3 = 5 \}$
- 27) $C = \{ m \mid m \text{ is a natural number, } m < 3 \}$
- 28) $C = \{ m \mid m \text{ is a natural number, } m \leq 3 \}$
- 29) $D = \{ m \mid m \in N, 2 < m < 5 \}$
- 30) $D = \{ m \mid m \in N, 2 \leq m < 5 \}$
- 31) $D = \{ m \mid m \in N, 2 < m \leq 5 \}$
- 32) $D = \{ m \mid m \in N, 3 < m < 7 \}$

Answers: 1) D equals the set of natural numbers greater than or equal to two.

3) A equals the set of natural numbers between 1 and 7 inclusive

5) D = the set of the seven dwarfs 7) V = the set of vowels

9) $D = \{x | x \in \mathbb{N}, x \geq 2\}$ 11) $A = \{x | x \text{ is a Natural number and } 1 \leq x \leq 7\}$

13) $P = \{x | x \text{ is a Natural number and } 3 \leq x \leq 11\}$

15) $P = \{x | x \text{ is a Natural number and } 3 < x < 11\}$

17) $A = \{5,6,7,8,9,10\}$ 19) $A = \{6,8,10,\dots\}$ 21) $A = \{5,6,7,\dots\}$

23) $A = \{ \}$ 25) $B = \{5\}$ 27) $C = \{1,2\}$ 29) $D = \{3,4\}$ 31) $D = \{3,4,5\}$