Section 1.4 Operations with sets – Union, Intersection and Complement

A **universal set** for a particular problem is a set which contains all the elements of all the sets in the problem.

A universal set is often denoted by a capital U, but sometimes the Greek letter ξ (xee) is used.

In this section we will create subsets of a given universal set and use set operations to create new subsets of the universal set.

There are three set operations we will learn in this section.

- **Complement:** The complement of a set A is symbolized by A' and it is the set of all elements in the universal set that are not in A.

- **Intersection:** The intersection of sets A and B is symbolized by $A \cap B$ and is the set containing all of the elements that are common to both set A and set B.

- **Union:** The union of set A and B is symbolized $A \cup B$ and is the set containing all the elements that are elements of set A or of set B or that are in both Sets A and B.

Here is a quick example to illustrate the 3 definitions.

Example: Let U be a universal set and A and B be subsets of U defined as follows.

U = {1,2,3,4,5}
A = {1,2,3}
B = {2,3,4}

Find A'

A' is all of the elements in the Universal set that are not in set A.

Answer: A' = {4,5}

Find $\boldsymbol{A \cap B}$ (This is asking me to find all of the elements that A and B have in common.)

Answer: $A \cap B$ = {2,3}

Find $\boldsymbol{A \cup B}$ (This is asking me to list all of the elements in A followed by all of the elements in B, then delete any elements that are written twice.)

$A \cup B$ = {1,2,3,2,3,4}

Answer: $A \cup B$ = {1,2,3,4}

Example: Let U be a universal set and A and B be subsets of U defined as follows.

U = {a,b,c,d,e,f}
A = {a,b,c}
B = {c,d,e}

Find $A' \cap B$

First I need to find A', which is all of the elements in U that aren't in set A.

A' = {d,e,f}

Now I can intersect the two sets.

$A' \cap B$ = {d,e,f} ∩ {c,d,e} (now find what the two sets have in common)

Answer: {d,e}

Find $A \cup B'$

First I need to find B'

B' = {a,b,f}

$A \cup B'$ = {a,b,c} ∪ {a,b,f} (put all 6 elements in a big set then delete the duplicates)

= {a,b,c,a,b,f}

Answer: {a,b,c,f}

#1-10: Find the following sets.

U = {a,b,c,d,e} A = {c,d,e} B = {a,c,d}

1) A'
2) B'
3) $A \cup B$
4) $A' \cup B'$
5) $A \cap B$
6) $A' \cap B'$
7) $A' \cap B$
8) $A \cap B'$
9) $A' \cup B$
10) $A \cup B'$

#11-20: Find the following sets.

U = {1,2,3,4,5} A = {1,2,3} B = {5}

11) A′ 12) B′

13) $A \cup B$ 14) $A' \cup B'$

15) $A \cap B$ 16) $A' \cap B'$

17) $A' \cap B$ 18) $A \cap B'$

19) $A' \cup B$ 20) $A \cup B'$

Example: Let U be a universal set and A, B and C be subsets of U defined as follows.

U = {a,b,c,d,e,f}
A = {a,b,c}
B = {c,d,e}
C = {d,e,f}

Find $\boldsymbol{A \cup B \cup C}$

I need to work from left to right. First I will find $A \cup B$

$A \cup B$ = {a,b,c} ∪ {c,d,e}
= {a,b,c,c,d,e}
= {a,b,c,d,e}

Now I can do the union C part. I can rewrite my problem as:

{a,b,c,d,e} ∪ C

= {a,b,c,d,e} ∪ {d,e,f}
= {a,b,c,d,e,d,e,f}

Answer: {a,b,c,d,e,f}

Find $(\boldsymbol{B} \cup \boldsymbol{C})'$

I have to work on the inside of the parenthesis first.

So I will first find:
$B \cup C$

$B \cup C$ = {c,d,e} ∪ {d,e,f}

$B \cup C$ = {c,d,e,d,e,f}

$B \cup C$ ={c,d,e,f}

Now I can do the complement.

I can replace the inside of the parenthesis with {c,d,e,f} and proceed to find its complement.

$(\mathrm{B} \cup \mathrm{C})'$ = (c,d,e,f)' (my answer will be all the elements of set U that are not in this set.)

Answer: {a,b}

Find $\boldsymbol{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})'$

First I need to simplify the parenthesis $(\mathrm{B} \cup \mathrm{C})'$
I just figured out that $(\mathrm{B} \cup \mathrm{C})'$ = {a,b}, so I will use the work I have already done

$\mathrm{A} \cup (\mathrm{B} \cup \mathrm{C})'$

= A ∪ {a, b}

= {a,b,c} ∪ {a,b}

= {a,b,c,a,b}

Answer: {a,b,c}

Find $A' \cap (B \cap C')$

I need to simplify the inside of the parenthesis first.

$(B \cap C')$

= {c,d,e} ∩ {a,b,c}

= {c}

$A' \cap (B \cap C')$

= A' ∩ $\{c\}$

= {d,e,f} ∩ {c}

Answer: Ø (empty set)

#21-32: Find the following sets.

U = {1,2,3,4,5,6} A = {1,2,3} B = {2,3,4} C = {1,5}

21) $A \cap C$ 22) $B \cap C$

23) $A \cup C$ 24) $B \cup C$

25) $A \cap B \cup C$ 26) $A \cup B \cap C$

27) $B \cup C \cap A$ 28) $B \cap A \cup C$

29) $A' \cap B$ 30) $A \cap B'$

31) $A' \cup B \cap C'$ 32) $B' \cap A \cup C'$

#33-44: Find the following sets.

U = {a,b,c,d} A = {a,b,c} B = {b,c,d} C = {a,d}

33) $A \cap C'$ 34) $B' \cap C$

35) $A' \cup C'$ 36) $B' \cup C'$

37) $A' \cap B \cup C'$ 38) $A' \cup B' \cap C$

39) $B' \cup C' \cap A$ 40) $B' \cap A' \cup C$

41) $A' \cap B'$ 42) $A \cap B'$

43) $A' \cup B' \cap C'$ 44) $B \cap A' \cup C'$

#45 – 56: Find the following sets.

U = {1,2,3,4,5,6} S = {2,4,6} T = {1,2,4} V = {4,5,6}

45) $S \cup (T \cap V)$

46) $(S \cup T) \cap V$

47) $(S \cup T)'$

48) $(V \cup S)'$

49) $S \cap (V \cap T')$

50) $(S' \cap V') \cup T$

51) $(S' \cup V') \cap T$

52) $S' \cup T \cap V'$

53) $T \cup V' \cup S'$

54) $T \cup V' \cap S'$

55) $(V \cap T)' \cup S$

56) $V \cup (S \cap T)'$

Answers: 1) {a,b} 3) {a,c,d,e} 5) {c,d} 7) {a} 9) {a,b,c,d} 11) {4,5} 13) {1,2,3,5} 15) ∅ 17) {5} 19) {4,5} 21) {1} 23) {1,2,3,5} 25) {1,2,3,5} 27) {1,2,3} 29) {4} 31) {2,3,4,6} 33) {b,c} 35) {b,c,d} 37) {b,c,d} 39) {a,b,c} 41) ∅ 43) ∅ 45) {2,4,6} 47) {3,5} 49) {6} 51) {1,2} 53) {1,2,3,4,5} 55) {1,2,3,4,5,6}