

### Section 3.5: The Mathematics of Finance - Loans

We will focus on loan problems in this section. There are formulas that can be used to solve each of the problems in this section. The formulas can get pretty messy. Fortunately the TI-83 and TI-84 calculators have the ability to solve loan problems without the need of using the messy formulas. The formulas are pre-programmed into the TI-83 and TI-84 calculator. We will just have to tell our calculator the unknowns, and what we are trying to find. Our calculator will plug the numbers in the correct formula and do any messy algebra that needs to be done.

Just to give you an idea of what loan formulas look like: Here are a two loan formulas.

<p>Loan payment formula:</p> $P = \frac{r(PV)}{1 - (1 + r)^{-n}}$ <p><i>P = Payment</i> <i>PV = Present Value</i> <i>r = rate per period</i> <i>n = number of periods</i></p>	<p>Length of time to pay off a loan:</p> $\text{Number of Periods} = \frac{\ln(\frac{FV}{PV})}{\ln(1 + r)}$ <p><i>FV = Future Value</i> <i>PV = Present Value</i> <i>r = rate per period</i></p>
---	--

We need to navigate to the screen where we will solve most of the loan problems in this section. Let's get to that screen by doing the following:

Calculator steps: Hit APPS button then select Finance then select TVM solver.

Your calculator should now display a screen with these variables. I have written what each variable stands for next to the variable. Your calculator likely has numbers next to some of the variables. This is okay. We will type over them when we need to solve a specific loan problem.

N	Number of payments
I%	Annual interest rate
PV	Loan amount (Normally entered as a <b><u>POSITIVE</u></b> number in loan problems.)
PMT	Monthly payment amount (Usually entered as a <b><u>NEGATIVE</u></b> number in loan problems.)
FV	Balance when loan is paid off. (Usually 0 in a loan problem.)
P/Y	Number of payments per year. (Usually 12 in a loan problem.)
C/Y	How often interest is charged. (Usually 12 in a loan problem)
Begin / End	Choose Begin if you make a loan payment the day you take your loan. Otherwise choose End. We will always leave this on END.

**Example:** A new car is purchased and a \$25,000 loan is taken. The loan is for 6 years (72 months) and the interest rate is 4.9% compounded monthly. What is the monthly payment?

Calculator steps: Hit APPS button then select Finance then select TVM solver.

We will solve most of the problems in this chapter from this screen. We need to know what each variable represents. We will enter values for every variable except for the one we are solving for. We are computing a monthly payment. We will put values in for every variable, except (PMT). PMT stands for the payment.

Enter these values on your calculator.

N (number of payments)	72
I% (annual interest rate)	4.9
PV (Loan amount, make this amount positive.)	25000
PMT	Move cursor here and hit green alpha button then enter after you have entered every other value.
FV (what you owe after make all payments)	0
P/Y (payments per year)	12
C/Y (how often interest is paid)	12
Begin / End  (We will always leave this on end.)	End

Once you have entered all of the amounts scroll up to PMT. Make sure your cursor is flashing next to PMT. It doesn't matter what value is in the PMT. Your calculator will put in the correct payment when you tell it to compute the payment. Hit the green ALPHA button the ENTER.

Answer: \$401.46

You may have noticed that the payment amount showed up as a negative number. This is correct. When a negative sign is in front of a number it means it is money you are paying.

The \$25,000 does not have a negative sign in front of it. You are not making a \$25,000 payment.

Homework #1-6:

- 1) A new car is purchased and a \$20,000 loan is taken. The loan is for 5 years (60 months) and the interest rate is 7.9% compounded monthly. What is the monthly payment?
- 2) A new car is purchased and a \$30,000 loan is taken. The loan is for 7 years (84 months) and the interest rate is 2.9% compounded monthly. What is the monthly payment?
- 3) A used car is purchased and a \$15,000 loan is taken. The loan is for 4 years (48 months) and the interest rate is 5.9% compounded monthly. What is the monthly payment?
- 4) A used car is purchased and a \$10,000 loan is taken. The loan is for 3 years (36 months) and the interest rate is 8% compounded monthly. What is the monthly payment?
- 5) \$150,000 is borrowed for the purchase of a new home. The loan is for 30 years (360 payments) and the interest rate is 4.5% compounded monthly. What is the monthly payment? (We are ignoring property taxes and insurance.)
- 6) \$200,000 is borrowed for the purchase of a new home. The loan is for 30 years (360 payments) and the interest rate is 4.5% compounded monthly. What is the monthly payment? (We are ignoring property taxes and insurance.)

**Example:** A new car is purchased and a \$25,000 loan is taken. The loan is for 6 years (72 months) and the interest rate is 4.9% compounded monthly. How much will be owed on the car after 2 years?

The TVM solver screen needs to have just calculated the payment for this loan to be able to figure out the balance after two years. This is the payment we calculated in the last example. Make sure you TVM solve screen has these amounts. The PMT should have a negative sign.

N	I	PV	PMT	FV	P/Y	C/Y	Begin End
72	4.9	25000	-401.46	0	12	12	End

Once the TVM solver has these amounts.

Hit 2<sup>nd</sup> and QUIT to get back to the main screen.

Hit the APPS button,

Select FINANCE,

Scroll down to option 9 and select BAL(

Your main screen should have BAL (

Two years represents 24 payments. Type 24 next to the balance and close the parenthesis. Then hit enter.

BAL(24)

Answer: \$17,467.12

Homework #7-12:

7) A new car is purchased and a \$20,000 loan is taken. The loan is for 5 years (60 months) and the interest rate is 7.9% compounded monthly. What is the balance after 3 years?

8) A new car is purchased and a \$30,000 loan is taken. The loan is for 7 years (84 months) and the interest rate is 2.9% compounded monthly. What is the balance after 3 years?

9) A used car is purchased and a \$15,000 loan is taken. The loan is for 4 years (48 months) and the interest rate is 5.9% compounded monthly. What is the balance after 1 year?

10) A used car is purchased and a \$10,000 loan is taken. The loan is for 3 years (36 months) and the interest rate is 8% compounded monthly. What is the balance after 1 year?

11) \$150,000 is borrowed for the purchase of a new home. The loan is for 30 years (360 payments) and the interest rate is 4.5% compounded monthly. What is the balance after 15 years?

12) \$200,000 is borrowed for the purchase of a new home. The loan is for 30 years (360 payments) and the interest rate is 4.5% compounded monthly. What is the balance after 10 years?

**Example:** A new car is purchased and a \$25,000 loan is taken. The loan is for 6 years (72 months) and the interest rate is 4.9%.

- a) What is the monthly payment?
- b) How much interest will be paid over the term of the loan?

N	I	PV	PMT	FV	P/Y	C/Y	Begin End
72	4.9	25000	Solve	0	12	12	End

Answer: Monthly payment \$401.46

b) Once the TVM solver has these amounts.

Hit 2<sup>nd</sup> and QUIT to get back to the main screen.

Hit the APPS button,

Select Finance,

Scroll down to option A and select:  $\sum Int($ .

Your main screen should have  $\sum Int($

I need to tell the calculator to compute the interest from the 1<sup>st</sup> payment to the last or 72<sup>nd</sup> payment. I do this like this:

$\sum Int( 1,72)$  (The 1 stands for the 1<sup>st</sup> payment and the 72 stands for the last payment.)

Answer: \$3905.46 (Notice the amount is negative on my calculator as it is money that is being spent.)

Homework #13-18:

13) A new car is purchased and a \$30,000 loan is taken. The loan is for 6 years (72 months) and the interest rate is 5.9% compounded monthly.

- a) What is the monthly payment?
- b) How much interest will be paid over the term of the loan?

14) A new car is purchased and a \$20,000 loan is taken. The loan is for 7 years (84 months) and the interest rate is 6% compounded monthly.

- a) What is the monthly payment?
- b) How much interest will be paid over the term of the loan?

15) A used car is purchased and a \$10,000 loan is taken. The loan is for 3 years (36 months) and the interest rate is 6% compounded monthly.

- a) What is the monthly payment?
- b) How much interest will be paid over the term of the loan?

16) A used car is purchased and a \$18,000 loan is taken. The loan is for 5 years (60 months) and the interest rate is 9% compounded monthly.

- a) What is the monthly payment?
- b) How much interest will be paid over the term of the loan?

17) \$200,000 is borrowed for the purchase of a new home. The loan is for 30 years (360 payments) and the interest rate is 4% compounded monthly.

- a) What is the monthly payment?
- b) How much interest will be paid over the term of the loan?

18) \$250,000 is borrowed for the purchase of a new home. The loan is for 30 years (360 payments) and the interest rate is 5.25% compounded monthly.

- a) What is the monthly payment?
- b) How much interest will be paid over the term of the loan?

**Example:** \$15,000 is still owed on a car loan. The loan has an interest rate of 5% compounded monthly and calls for monthly payments. The current payment is \$250 per month. How long will it take to pay the loan off? Round to the nearest month.

Enter these amounts and move your cursor to N and hit ALPHA and ENTER to solve for N.

N	I	PV	PMT	FV	P/Y	C/Y	Begin End
ALPHA ENTER	5	15000 (amount owed, needs to be positive)	-250 (needs to be negative)	0	12	12	End

$N = 69.187$

Answer: about 69 months. (It will take 69 payments of \$250 and then one small payment. We can figure out the amount that will still be owed after the 69<sup>th</sup> payment. Do this by finding BAL(69). I won't ask for this on a test.)



Homework: 19 - 24

19) \$10,000 is still owed on a car loan. The loan has an interest rate of 2.9% compounded monthly and calls for monthly payments. The current payment is \$200 per month. How long will it take to pay the loan off? Round to the nearest month.

20) \$25,000 is still owed on a car loan. The loan has an interest rate of 7.9% compounded monthly and calls for monthly payments. The current payment is \$450 per month. How long will it take to pay the loan off? Round to the nearest month.

21) \$15,500 is still owed on a car loan. The loan has an interest rate of 6% compounded monthly and calls for monthly payments. The current payment is \$300 per month. How long will it take to pay the loan off? Round to the nearest month.

22) \$8,500 is still owed on a car loan. The loan has an interest rate of 4% compounded monthly and calls for monthly payments. The current payment is \$250 per month. How long will it take to pay the loan off? Round to the nearest month.

23) \$150,000 is still owed on a home loan. The loan has an interest rate of 4.5% compounded monthly and calls for monthly payments. The current payment is \$1,100 per month. How long will it take to pay the loan off? Round to the nearest month.

24) \$200,000 is still owed on a home loan. The loan has an interest rate of 6.25% compounded monthly and calls for monthly payments. The current payment is \$1200 per month. How long will it take to pay the loan off? Round to the nearest month.

**Example:** Anthony can afford \$350 per month for a truck payment. What size of a truck loan can he take if he plans to finance at:

a) 6 years (72 months) at an interest rate of 5% compounded monthly?

b) 5 years (60 months) years at an interest rate of 5% compounded monthly?

The N will be the number of payments.

I need to solve for PV in each part of this question.

I will enter the desired payment in for PMT. (This should be entered as a negative.)

The FV will be 0 as the truck will have to be paid off at the end of the loan.

Both the P/Y and C/Y will be 12 as monthly payments are being made.

a)

Enter these amounts and move your cursor to N and hit ALPHA and ENTER to solve for N.

N	I	PV	PMT	FV	P/Y	C/Y	Begin End
72	5	Alpha solve	-350 (needs to be negative)	0	12	12	End

Answer: \$21,732.47

b)

Enter these amounts and move your cursor to N and hit ALPHA and ENTER to solve for N.

N	I	PV	PMT	FV	P/Y	C/Y	Begin End
60	5	Alpha solve	-350 (needs to be negative)	0	12	12	End

Answer: \$18,546.75

Homework: 25 – 31

25) Anthony can afford \$250 per month for a truck payment. What size of a truck loan can he take if he plans to finance at:

- a) 6 years (72 months) at an interest rate of 6% compounded monthly?
- b) 5 years (60 months) years at an interest rate of 6% compounded monthly?

26) Sarah can afford \$400 per month for a car payment. What size of a car loan can she take if he plans to finance at:

- a) 6 years (72 months) at an interest rate of 2.9% compounded monthly?
- b) 7 years (84 months) years at an interest rate of 2.9% compounded monthly?

27) Kris can afford \$475 per month for a car payment. What size of a car loan can she take if he plans to finance at:

- a) 6 years (72 months) at an interest rate of 6% compounded monthly?
- b) 7 years (84 months) years at an interest rate of 6% compounded monthly?

28) Lisa can afford \$300 per month for a truck payment. What size of a truck loan can she take if he plans to finance at:

- a) 6 years (72 months) at an interest rate of 2.9% compounded monthly?
- b) 7 years (84 months) years at an interest rate of 2.9% compounded monthly?

29) Jack and Jill can afford \$1200 per month for a house payment. What size of a home loan can the take if they plan to finance for 30 years (360 months) at 4.5% compounded monthly? (This ignores property taxes and insurance that will likely be part of the house payment. I don't want to over-complicate this problem.)

30) Herman and Lilly can afford \$900 per month for a house payment. What size of a home loan can the take if they plan to finance for 30 years (360 months) at 4.25% compounded monthly?

31) Jethro and Ellie can afford \$750 per month for a house payment. What size of a home loan can the take if they plan to finance for 30 years (360 months) at 5.25% compounded monthly?

## Answers

1) Answer: \$404.57

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
60	7.9	20000	solve	0	12	12	End

3) Answer: \$351.59

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
48	5.9	15000	solve	0	12	12	End

5) Answer: \$760.03

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
360	4.5	150000	solve	0	12	12	End

7) Make sure your TVM solve looks like problem 1.  
BAL(36)

Answer: \$8954.32

9) Make sure your TVM solve looks like problem 3.  
BAL(12)

Answer: \$11,574.29

11) Make sure your TVM solve looks like problem 5.  
BAL(180)

Answer: \$99,350.93

13a) Answer: \$495.77

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
72	5.9	30000	solve	0	12	12	End

13b)  $\sum Int(1,72)$

Answer: \$5695.56

15a) Answer: \$304.22

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
36	6	10000	solve	0	12	12	End

15b)  $\sum Int(1,36)$

Answer: \$951.90

17a) Answer: \$954.83

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
360	4	200000	solve	0	12	12	End

17b)  $\sum Int(1,360)$

Answer: \$143,739.01

19) Answer: (53.36 months) 53 full payments and one partial payment

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
Solve	2.9	10000	<b>-200</b>	0	12	12	End

21) Answer: (59.92 months) 59 full payments and one partial payment

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
Solve	6	15500	<b>-300</b>	0	12	12	End

23) Answer: 191.33 payments (191 full payments and one partial payment) 191 months is 15 years and 11 months.

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
Solve	4.5	150000	<b>-1100</b>	0	12	12	End

25a) Answer: \$15,084.88

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
72	6	solve	-250	0	12	12	End

25b) Answer: \$12,931.39

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
60	6	solve	-250	0	12	12	End

27a) Answer: \$28,661.27

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
72	6	solve	-475	0	12	12	End

27b) Answer: \$32,515.20

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
84	6	solve	-475	0	12	12	End

29) Answer: \$236,833.39

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
360	4.5	solve	-1200	0	12	12	End

31) Answer: \$135,819.44

N	I	PV	PMT	FV	P/Y	C/Y	Begin / End
360	5.25	solve	-750	0	12	12	End

