

Section 4.3: Tree Diagrams and the Counting Principle

It is often necessary to know the total number of outcomes in a probability experiment. The Counting Principle is a formula that allows us to determine the number of possible outcomes in a probability experiment without having to create the sample space.

The Counting Principle:

“If a first experiment can be performed in M distinct ways and a second experiment can be performed in N distinct ways then the two experiments in that specific order M first followed by N can be performed in $M \times N$ distinct ways”

Example: Use the Counting Principle to find the number of possible outcomes of these two “experiments” done in this specific order:

Experiment 1: a coin is tossed and the side that is face up is recorded

Experiment 2: a die is rolled and the number that is face up is recorded

Tossing a coin has 2 possible outcomes...H or T

Rolling a die has 6 possible outcomes... 1, 2, 3, 4, 5, 6

The Counting Principle tells me that the total of outcomes of experiment 1 followed by experiment 2 can be found by multiplying the number of ways each experiment can happen.

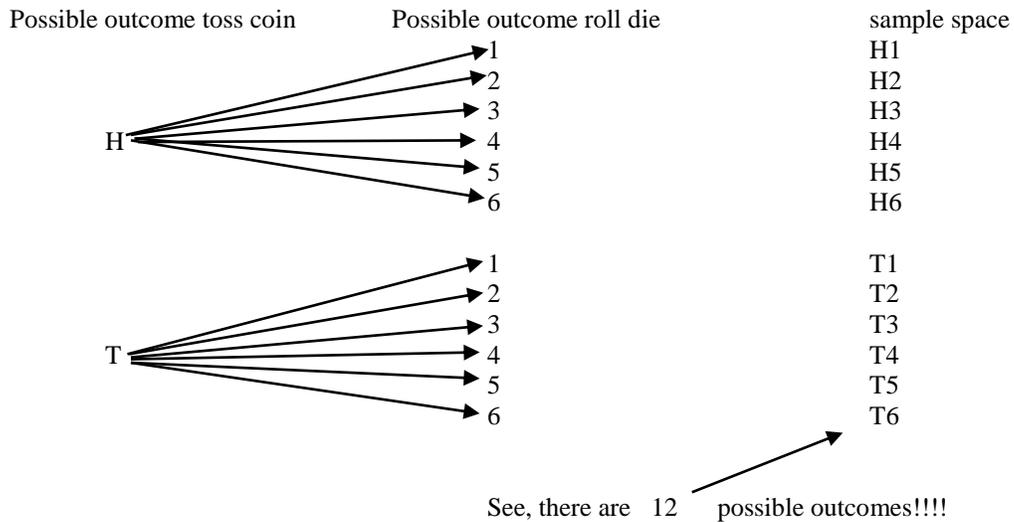
The two experiments together tossing a coin then rolling a dice have $2 \times 6 = 12$ possible outcomes.

Answer: 12 ways

A tree diagram is an effective way to find all of the elements in a sample space. The experiment we are talking about has 12 possible outcomes when they are done in the order described. It is often helpful to list all of the possible outcomes.

Example: Use a “Tree Diagram” to create the “sample space” for this situation. That is create a “sample space” of the 12 possible outcomes.

I will better explain how create a tree diagram in my video. Here is the result.



Answer: Sample Space: $S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

Example: Two Balls are to be selected without replacement from a bag that contains one red, one blue, one green and one orange ball.

A) Use the counting principle to determine the number of possible points in the sample space.

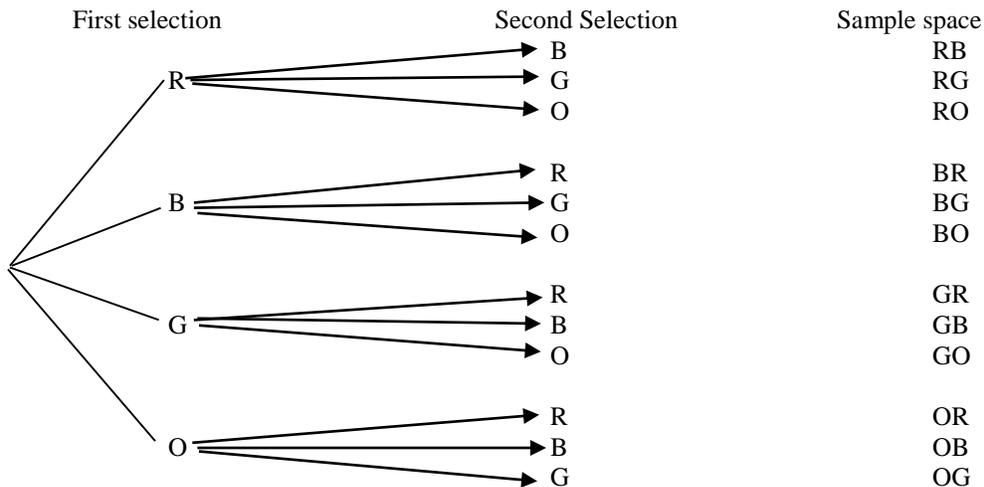
The first experiment is selecting a ball from the bag that has 4 balls.

The second experiment is selecting a ball from the 3 balls that are left in the bag after the first ball is removed. (The words without replacement tells me the ball that was selected won't be put back)

$$\begin{array}{ccc} & \frac{4}{\uparrow} \times \frac{3}{\uparrow} = 12 \text{ points in the sample space} & \\ \# \text{ of choices for 1}^{\text{st}} \text{ ball} & & \# \text{ of choices for 2}^{\text{nd}} \text{ ball} \end{array}$$

Answer: 12

B) Construct a tree diagram and list the sample space.



Answer: Sample space = {RB, RG, RO, BR, BG, BO, GR, GB, GO, OR, OB, OG}

C) Determine the probability that an orange ball is selected.

I need to create a fraction.

The numerator needs to be the number of elements in the sample space that have an orange ball (RO BO GO OR OB OG). The numerator will be 6.

The denominator will be the 12 elements in the sample space.

$$P(\text{one orange ball selected}) = \frac{\text{\# of occurrences of } O}{\text{total \# of outcomes in sample space}} = \frac{6}{12} = \frac{1}{2}$$

Answer: $\frac{1}{2}$

Example: Two Balls are to be selected with replacement from a bag that contains one red, one blue, one green and one orange ball. Use the counting principle to determine the number of possible points in the sample space.

The first experiment is selecting a ball from the bag that has 4 balls.

The second experiment will also involve selecting a ball from a bag that has 4 balls. The words with replacement tell me the ball is put back in the bag before the second ball is selected.

$$\frac{4}{\text{\# of choices for 1st ball}} \times \frac{4}{\text{\# of choices for 2nd ball}} = 16 \text{ points in the sample space}$$

Answer: 16

Example: Joe has 7 shirts, 4 pairs of pants and 2 pairs of shoes. He needs to make an outfit containing one of each item. How many different outfits are possible? (We assume that one different item makes a different outfit)

This is a Counting Principle problem. It has three “experiments” as opposed to 2. The Counting Principle can be extended to any number of experiments.

Experiment 1: select a shirt (7 possible outcomes)

Experiment 2: select a pair of pants (4 possible outcomes)

Experiment 3: select a pair of shoes (2 possible outcomes)

Answer: $7 \cdot 4 \cdot 2 = 56$ different outfits.

Homework #1-19: Use the Counting Principle to answer the following questions.

1) If two dates are selected at random from the 365 days of the year and are listed in the order they were selected.

a) Use the counting principle to determine the number of possible outcomes if the dates are selected **with replacement**.

b) Use the counting principle to determine the number of possible outcomes if the dates are selected **without replacement**.

2) Two dates are selected at random from the month of May and listed in the order they were selected.

a) Use the counting principle to determine the number of possible outcomes if the dates are selected **with replacement**.

b) Use the counting principle to determine the number of possible outcomes if the dates are selected **without replacement**.

3) Joe has 5 shirts, 3 pairs of pants and 4 pairs of shoes. He needs to make an outfit containing one of each item. How many different outfits are possible? (We assume that one different item makes a different outfit)

4) Shannon is going to purchase a new car. The car has 5 exterior color choices, 2 interior color packages and 2 engine options. How many different cars are possible?

5) A television remote has buttons for the digits 0 – 9. If you press two buttons, how many numbers are possible if

a) The same button may be pressed twice?

b) The same button may not be pressed twice?

6) A safe's lock has a three number combination. The numbers on the safe lock are 0 – 36. How many combinations are possible if no number may be used more than once?

7) A movie theater sells 3 sizes of popcorn (small, medium, and large) with 3 choices of toppings (no butter, butter, extra butter). How many possible ways can a bag of popcorn be purchased?

8) Girls' ice skates come with the following options:

Colors: white, beige, pink, yellow, blue

Sizes: 4, 5, 6, 7, 8

Extras: tassels, striped laces, bells

Assuming that all skates are sold with ONE extra, how many possible arrangements exist?

9) A state issues license plates consisting of letters and numbers. There are 26 letters and the letters may be repeated. There are 10 digits and the digits may be repeated. How many possible license plates can be issued with two letters followed by three numbers?

10) An ice cream shop offers 31 flavors. You order a double-scoop cone. In how many different ways can the clerk put the ice cream on the cone if you wanted two different flavors?

11) Burger Queen offers 4 types of burgers, 5 types of beverages, and 3 types of desserts. If a meal consists of 1 burger, one beverage and one dessert, how many possible meals can be chosen?

12) Elizabeth is choosing silverware with which to eat dinner. There are 3 knives and 2 forks to choose from. How many different silverware sets can Elizabeth choose?

13) Reid is ordering a birthday cake for a friend. There are 5 cake flavors and 4 frosting flavors to choose from. How many different cakes can Reid order?

14) If car license plates consists 3 letters followed by 3 numbers, how many license plates are possible if there are no restrictions?

15) Answer question 14 but assume the license plates must be of the form 2 letters followed by 4 numbers and there are no other restrictions.

16) The standard New York state license plate has three letters followed by four digits. How many different license plates are possible if the digits can't be repeated and letters can be repeated?

17) The standard New York state license plate has three letters followed by four digits. How many different license plates are possible if neither the digits nor the letters can be repeated?

18) Students at a small college are assigned student identification numbers. The IDs are made up of five digits. If the digits can be repeated, how many identification numbers are possible?

19) You are assigned a computer generated 4-digit password to access your new voice mail account. If the digits can be repeated, how many passwords are possible?

#20 – 30: Counting principle problems with sample space construction and probability questions.

20) A couple plans to have three children.

- a) Determine the number of points in the sample space of the possible arrangements of boys and girls.
- b) Construct a tree diagram and list the sample space.

Find the probability the couple has:

- c) Two girls
- d) At least one girl
- e) A girl then two boys

21) A couple plans to have two children.

- a) Determine the number of points in the sample space of the possible arrangements of boys and girls.
- b) Construct a tree diagram and list the sample space.

Find the probability the couple has:

- c) Two girls
- d) At least one girl
- e) A girl then a boy

22) Answer each part of question 21, but assume the family plans to have 4 children.

23) A coin is tossed three times and a sequence of heads and tails is recorded.

- a) Determine the number of points in the sample space
- b) Construct a tree diagram and list the sample space

Find the probability that:

- c) No heads are tossed
- d) Exactly one head is tossed
- e) Three heads are tossed

24) Answer each part of question 23 but assume a coin is tossed two times.

25) Two six sided dice are rolled and the number on each face is recorded.

- a) Determine the number of points in the sample space
- b) Construct a tree diagram and list the sample space
- c) Find the probability that a double is rolled (both dice have the same number)
- d) Find the probability that a sum of 7 is rolled
- e) Find the probability a sum of 2 is rolled
- f) Are you as likely to roll a sum of 2 as you are of rolling a sum or 7

26) Answer each part of question 25 but assume the first dice is a 5 sided dice and the second dice is a four sided dice.

27) A coin is flipped then a 6 sided dice is rolled.

- a) Determine the number of points in the sample space
- b) Construct a tree diagram and list the sample space
- c) Find the probability that a head is flipped and an even number is rolled
- d) Find the probability that a tail is flipped and a number less than 3 is rolled

28) Answer question 27 but assume that a coin is flipped and a 5 sided dice is rolled.

29) A coin is flipped then a number is picked out of a hat containing the numbers 1,2 and 3.

- a) Determine the number of points in the sample space
- b) Construct a tree diagram and list the sample space
- c) Find the probability that a head is flipped and an even number is selected
- d) Find the probability that a tail is flipped and a number less than 3 is selected

30) Answer question 29 but assume that a coin is flipped and a number is picked out of a hat containing the numbers 1,2,3 and 4.

Answers:

- 1a) $365 \cdot 365 = 133,225$ ways 1b) $365 \cdot 364 = 132860$ ways
3) $5 \cdot 3 \cdot 4 = 60$ outfits 5a) $10 \cdot 10 = 100$ 5b) $10 \cdot 9 = 90$
7) $3 \cdot 3 = 9$ 9) $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 676,000$
11) $4 \cdot 5 \cdot 3 = 60$ 13) $5 \cdot 4 = 20$
15) $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6,760,000$ 17) $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 78,624,000$
19) $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$
21a) $2 \cdot 2 = 4$ 21b) {BB BG GB GG} 21c) $\frac{1}{4}$ 21d) $\frac{3}{4}$ 21e) $\frac{1}{4}$
23a) $2 \cdot 2 \cdot 2 = 8$ 23a) {HHH HHT HTH HTT THH THT TTH TTT}
23c) $\frac{1}{8}$ 23d) $\frac{3}{8}$ 23e) $\frac{1}{8}$ 25a) $6 \cdot 6 = 36$
25b) (1,1) sum 2 (1,2) sum 3 (1,3) sum 4 (1,4) sum 5 (1,5) sum 6 (1,6) sum 7
(2,1) sum 3 (2,2) sum 4 (2,3) sum 5 (2,4) sum 6 (2,5) sum 7 (2,6) sum 8
(3,1) sum 4 (3,2) sum 5 (3,3) sum 6 (3,4) sum 7 (3,5) sum 8 (3,6) sum 9
(4,1) sum 5 (4,2) sum 6 (4,3) sum 7 (4,4) sum 8 (4,5) sum 9 (4,6) sum 10
(5,1) sum 6 (5,2) sum 7 (5,3) sum 8 (5,4) sum 9 (5,5) sum 10 (5,6) sum 11
(6,1) sum 7 (6,2) sum 8 (6,3) sum 9 (6,4) sum 10 (6,5) sum 11 (6,6) sum 12
25c) $\frac{6}{36} = \frac{1}{6}$ 25d) $\frac{6}{36} = \frac{1}{6}$ 25e) $\frac{1}{36}$ 25f) no 7 is more likely

27a) $2 \cdot 6 = 12$
27b) {H1 H2 H3 H4 H5 H6 T1 T2 T3 T4 T5 T6}
27c) $\frac{3}{12} = \frac{1}{4}$ 27d) $\frac{2}{12} = \frac{1}{6}$
29a) $2 \cdot 3 = 6$ 29b) {H1 H2 H3 T1 T2 T3}
29c) $\frac{1}{6}$ 29d) $\frac{2}{6} = \frac{1}{3}$